SYLLABUS DEVELOPMENT GUIDE

AP® Precalculus

The guide contains the following information:

Curricular Requirements

The curricular requirements are the core elements of the course. A syllabus must provide explicit evidence of each requirement based on the required evidence statement(s). The Unit Guides and the "Instructional Approaches" section of the *AP® Precalculus Course and Exam Description* (CED) may be useful in providing evidence for satisfying these curricular requirements.

Required Evidence

These statements describe the type of evidence and level of detail required in the syllabus to demonstrate how the curricular requirement is met in the course.

Note: Curricular requirements may have more than one required evidence statement. Each statement must be addressed to fulfill the requirement.

Samples of Evidence

For each curricular requirement, two to three separate samples of evidence are provided. These samples provide either verbatim evidence or clear descriptions of what acceptable evidence could look like in a syllabus. In some samples, the specific language that addresses the required evidence is highlighted in **bold** text.

Curricular Requirements

CR1 The students and teacher have access to a college-level precalculus textbook, in print or electronic format.

CR2 The course is structured to incorporate the required content (Units 1, 2, and 3) outlined in each of the units described in the AP Precalculus Course and Exam Description.

CR3 The course provides opportunities for students to develop the skills related to Mathematical Practice 1: Procedural and Symbolic Fluency.

CR4 The course provides opportunities for students to develop the skills related to Mathematical Practice 2: Multiple Representations.

CR5 The course provides opportunities for students to develop the skills related to Mathematical Practice 3: Communication and Reasoning.

CR6 Students have access to graphing calculators and opportunities to use them to solve problems and to explore and interpret precalculus concepts.

CR7 The course provides opportunities for students to practice building function models in settings where students must choose, construct, and defend the selection of a function model.

Curricular Requirement 1

The students and teacher have access to a college-level precalculus textbook, in print or electronic format.

Required Evidence

- ☐ The teacher must provide the title, author, and publication date of a college-level precalculus textbook.

Samples of Evidence

1. The teacher selects a preapproved college-level textbook.
2. The teacher provides the title, author, and publication date of a college-level precalculus textbook.

Curricular Requirement 2

The course is structured to incorporate the required content (Units 1, 2, and 3) outlined in each of the units described in the AP Precalculus Course and Exam Description.

Required Evidence

- ☐ The syllabus must include an outline of course content by unit topic or collection of topics using any organizational approach.

  Note: If the syllabus demonstrates a different approach than the units outlined in the AP Precalculus Course and Exam Description (CED), the teacher must indicate where the content in the CED will be taught.

Samples of Evidence

1. Our course divides the content into the following units:
   1. Polynomial Functions (CED Unit 1)
   2. Rational Functions (CED Unit 1)
   3. Exponential Functions (CED Unit 2)
   4. Logarithmic Functions (CED Unit 2)
   5. Trigonometric Functions (CED Unit 3)
   6. Polar Functions (CED Unit 3)

2. We will cover the following sections:

   Chapter 1 Linear Functions and Change (CED Unit 1)

   1.1 Functions and Function Notation
   1.2 Rate of Change
   1.3 Linear Functions
   1.4 Formulas for Linear Functions
   1.5 Graphs and Models with Linear Functions and Inequalities
   1.6 Fitting Linear Functions to Data

Chapter 2 Functions (CED Unit 1)

2.1 Input and Output
2.2 Domain and Range
2.3 Piecewise-Defined Functions
2.4 Preview of Transformations: Shifts
2.5 Preview of Composite and Inverse Functions
2.6 Concavity

Chapter 3 Quadratic Functions (CED Unit 1)

3.1 Introduction to the Family of Quadratic Functions
3.2 The Vertex of a Parabola

Chapter 4 Exponential Functions (CED Unit 2)

4.1 Introduction to the Family of Exponential Functions
4.2 Comparing Exponential and Linear Functions
4.3 Graphs of Exponential Functions
4.4 Applications to Compound Interest
4.5 The Number *e*

Chapter 5 Logarithmic Functions (CED Unit 2)

5.1 Logarithms and Their Properties
5.2 Logarithms and Exponential Models
5.3 The Logarithmic Function and Its Applications
5.4 Logarithmic Scales

Chapter 6 Transformations of Functions and Their Graphs (CED Unit 3)

6.1 Shifts, Reflections, and Symmetry
6.2 Vertical Stretches and Compressions
6.3 Horizontal Stretches and Combinations of Transformations

Chapter 7 Trigonometry Starting with Circles (CED Unit 3)

7.1 Introduction to Periodic Functions
7.2 The Sine and Cosine Functions
7.3 Radians and Arc Length
7.4 Graphs of Sine and Cosine Functions
7.5 Sinusoidal Functions
7.6 The Tangent Function
7.7 The Six Trigonometric Functions and Relationships Between Them
7.8 Inverse Trigonometric Functions

Chapter 9 Trigonometric Identities, Polar Coordinates, and Complex Numbers (CED Unit 3)

9.1 Trigonometric Equations
9.2 Identities, Expressions, and Equations
9.3 Sum and Difference Formulas for Sine and Cosine
9.4 Polar Coordinates
9.5 Complex Numbers and De Moivre's Theorem

Chapter 11 Polynomial and Rational Functions (CED Unit 1)

11.1 Power Functions and Proportionality
11.2 Polynomial Functions and Their Behavior
11.3 Zeros of Polynomials and Short-Run Behavior
11.4 Rational Functions
11.5 The Short-Run Behavior of Rational Functions
11.6 Comparing Power, Exponential, and Log Functions
11.7 Fitting Exponential and Polynomials to Data

Chapter 12 Vectors and Matrices (CED Unit 4)

12.1 Vectors

12.2 The Components of a Vector

12.3 Applications of Vectors

12.4 The Dot Product

12.5 Matrices

Chapter 14 Parametric Equations and Conic Sections (CED Unit 4)

14.1 Parametric Equations

14.2 Implicitly Defined Curves and Circles

14.3 Ellipses

14.4 Hyperbolas

14.5 Geometric Properties of Conic Sections

14.6 Hyperbolic Functions

3. The submitted AP Precalculus syllabus contains a list of the following units listed in the AP Precalculus CED:
    1. Unit 1: Polynomial and Rational Functions
    2. Unit 2: Exponential and Logarithmic Functions
    3. Unit 3: Trigonometric and Polar Functions
    4. Unit 4: Functions Involving Parameters, Vectors, and Matrices

Curricular Requirement 3

The course provides opportunities for students to develop the skills related to Mathematical Practice 1: Procedural and Symbolic Fluency.

Required Evidence

- ☐ The syllabus must include a description or copy of at least one lesson, activity, or assignment in which students are algebraically manipulating functions expressed analytically, without technology.

Samples of Evidence

1. A homework assignment asks students to **solve by hand** a variety of equations and inequalities involving polynomial functions.
2. On a worksheet, students are presented with the functions $y=2x^2-3x+2$ and $y=64x-2$. Students are asked to **locate the intersection point without technology** and then graph the two functions, choosing an appropriate scale. Students are then asked to shade the region determined by the inequality $2x^2-3x+2<y<64x-2$.
3. In a homework assignment, students will **rewrite** rational functions in a form that allows them to determine the function's removable and non-removable discontinuities **without technology**. In class the next day, they will work in partners to answer the question: When is it helpful to rewrite a rational function in an equivalent form, and when is it not? What forms of rational functions are most helpful to determine certain essential characteristics of the function (such as end behavior, discontinuities, etc.)?

Curricular Requirement 4

The course provides opportunities for students to develop the skills related to Mathematical Practice 2: Multiple Representations.

Required Evidence

- ☐ The syllabus must include a description or copy of at least one lesson, activity, or assignment in which students work with multiple representations. Each of the four representations (analytical, numerical, graphical, and verbal) must be in at least one of the provided lessons, activities, or assignments. It is not necessary that all four representations appear in a single lesson, activity, or assignment.

  AND

  There must be evidence of a connection between at least two different representations in at least one of the provided lessons, activities, or assignments.

Clarifying Terms

Representations – Analytical: one that is given as an algebraic expression or equation.

Representations – Numerical: one whose values are given as tabular data or sets of ordered pairs.

Representations – Graphical: one that is drawn as a set of points on a pair of coordinate axes or in which information is presented in a diagram.

Representations – Verbal: one that is described in words rather than mathematical symbols.

Samples of Evidence

1. For a homework assignment, students are given a **table of values** showing the cost in dollars for installing x square meters of carpet in an office. They must then write an **analytical expression** for the cost as a function for the number of square meters installed.

   An exam question gives students **verbal descriptions** of various functions (such as "The function f is positive, decreasing, and concave down") and asks them to **construct** a graph for a function **matching** each verbal description.
2. In a classroom activity, groups of students are each given a set of 24 index cards. The 24 shuffled cards consist of six **sets of the graphical, numerical, verbal, and analytic representations** of six exponential functions. In the groups, students **sort** the cards into the six functions in four representations.
3. In a culminating end-of-week performance task, students will work individually to model population growth in cities in the United States, South America, or Southeast Asia **using equivalent exponential analytical, numerical, graphical, and verbal representations**. Then, they will present their models to a small group, draw **connections** between the models, and draw conclusions about the population patterns in the different cities using the models.

Curricular Requirement 5

The course provides opportunities for students to develop the skills related to Mathematical Practice 3: Communication and Reasoning.

Required Evidence

- ☐ The syllabus must include a description or copy of at least one lesson, activity, or assignment in which students use appropriate mathematical language while providing rationales for conclusions.

Samples of Evidence

1. In a group activity, students are given a table of values of a population as a function of time in which the time intervals are of varying lengths; the students compute the average rate of change of the population on each interval and apply their results to **refute or support** the claim that the population is increasing at an increasing rate. The teacher walks around the room ensuring that each student is using **appropriate mathematical language** in making their arguments.
2. In a classroom task, pairs of students are each given three functions and three scenarios, each pair of which represents the temperature of an object as a function of time. The functions and scenarios are not given in pairs to the students, so the students' task is to determine which function corresponds to which scenario. Students will present a function-scenario match and, **using appropriate mathematical language, provide a rationale** for the match by identifying features of the function and scenario.
3. In a homework assignment, students will **craft explanations using appropriate mathematical language** for which models of a dataset will reach a certain threshold value first. The following day in class, students will trade their justifications with a partner and evaluate the rationale against a Criteria for Success (rubric) before getting feedback from the teacher against that same rubric.

Curricular Requirement 6

Students have access to graphing calculators and opportunities to use them to solve problems and to explore and interpret precalculus concepts.

Required Evidence

- ☐ The syllabus includes a statement that each student has individual access to an approved graphing calculator.

  AND

  The syllabus must include a description or copy of lessons, activities, or assignments in which students use graphing calculators to do each of the following:

  1. Identify minima/maxima of functions
  2. Identify numerical solutions to equations in one variable
  3. Find regression equations to model data and analyze residual graphs

  It is not necessary that the requirements appear in a single lesson, activity, or assignment.

Samples of Evidence

1. I have a **classroom set of graphing calculators** that are sufficient in number to guarantee that each student who does not own a graphing calculator will be able to borrow one for the year.

   A worksheet gives students practice problems that cannot be solved analytically in which they must use their calculators to find the roots of functions, find where two functions intersect, **find the extrema of functions**, and **find solutions to equations**; all calculator tools used must give answers with the number of correct decimal places required on the AP Exam (so, for example, students should use the built-in equation solver to solve equations rather than graphing and tracing).

   In a group project, students collect their own data from some real-world situation and attempt to choose an appropriate model (such as linear or exponential) for the data. They then use the calculator to **compute a regression** for the data, and finally they use the calculator to graph the **residuals** in order to attempt to determine if their chosen model was appropriate.

2. Each student will need to **purchase a graphing calculator** to use in class and at home; the school will **provide a graphing calculator** to any student who cannot purchase one. In an activity, students are presented with the function f(x)=x^4-4x^3+2. They are asked to **use the calculator** to **find the zeros** of this function and any **local maxima or minima**. They are then asked to find the **intersection points** of f(x) with the function g(x)=2x-2.

   As an assignment, students are given a table of values for a function H(t) that represents the height of a tree as a function of time in years. They are asked to make a scatter plot and use the calculator to **find the regression equation** for this data and analyze the residuals.

3. Students **have access to a classroom set of graphing calculators** that can be checked out and used for the duration of class, as well as to complete homework.

   Students are given an instructional activity in which they are presented with a logarithmic and exponential function that are inverses (the inverse relationship is not revealed). Using the calculator, they complete a guided inquiry in which they **identify the intercepts, search for extrema, and determine points of intersection** of the two functions to discover the inverse relationship between the exponential and logarithmic functions.

   In a classroom activity, students assume the title of social media manager for the day and leverage a graphing calculator to **model data** about the spread of a TikTok video using both a quadratic and exponential regression. Then, they **analyze residual graphs** to determine the most accurate model for the data set.

Curricular Requirement 7

The course provides opportunities for students to practice building function models in settings where students must choose, construct, and defend the selection of a function model.

Required Evidence

☐ The syllabus must include a description or copy of at least one lesson, activity, or assignment in which students are given mathematical or contextual criteria such as zeros, end behavior, and covariation of quantities and are asked to select and build a model satisfying the criteria (without using regression) and provide a rationale for their selection.

AND

A description or copy of at least one lesson, activity, or assignment in which students are given bivariate data and are asked to select a model for the data and build the model using regression and then assess the suitability of their selection using residual plots.

In both cases, students should not be given direction toward a specific function model. The criteria and data sets can be supplied by the teacher or be self-selected.

Samples of Evidence

1. An exam question gives students information on some cyclic phenomena, including maximal and minimal values and time between these values. Students must **write formulas** for functions that model these phenomena and **justify** how their functions meet the given criteria.

   In a group project, students collect their own data from some real-world situation and try to **choose an appropriate model** (such as linear or exponential) for the data. They then use the calculator to **compute a regression** for the data, and finally they use the calculator to graph the residuals in order to attempt to **determine if their chosen model was appropriate**.

2. On a worksheet, students are presented with 6–8 tables of values that result from different increasing linear, exponential, and logarithmic functions. Students are not told what type of function corresponds to each table. Students work in groups to **decide** what type of function corresponds to each table, providing a **rationale** based on how output values are changing with respect to input values. After verifying the function type with the class, students **build** the appropriate functions.

   On a worksheet, students are presented with tables of values for three functions. For each table, students must **determine an appropriate model** for each function by calculating regressions and analyzing the residual plots. Students will present their solutions to the class and **explain their reasoning**.

3. In a homework assignment, students are provided with characteristics of a “mystery function” such as the number and type of zeros, the end behavior, and the locations of extrema. They **build** a function model from those criteria, without using regressions, and **craft a justification** for their model. The following day in class, they compare their models and justifications with their classmates’ models and justifications, and the class **decides** on the strongest model of the “mystery function.”

   In a classroom activity, students leverage a graphing calculator to **model a data set using linear, quadratic, and exponential regressions.** Then, they **analyze residual graphs** to determine the most accurate model for the data set.