PROFESSIONAL DEVELOPMENT

AP® Physics 2

The Capacitor as a Bridge from Electrostatics to Circuits

CURRICULUM MODULE

For the redesigned course launching fall 2014

The College Board
New York, NY

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Preface

AP® curriculum modules are exemplary instructional units composed of one or more lessons, all of which are focused on a particular curricular topic; each lesson is composed of one or more instructional activities. Topics for curriculum modules are identified because they address one or both of the following needs:

- A weaker area of student performance as evidenced by AP Exam subscores
- Curricular topics that present specific instructional or learning challenges

The components in a curriculum module should embody and describe or illustrate the plan/teach/assess/reflect/adjust paradigm:

1. *Plan* the lesson based on educational standards or objectives and considering typical student misconceptions about the topic or deficits in prior knowledge.
2. *Teach* the lesson, which requires active teacher and student engagement in the instructional activities.
3. *Assess* the lesson, using a method of formative assessment.
4. *Reflect* on the effect of the lesson on the desired student knowledge, skills, or abilities.
5. *Adjust* the lesson as necessary to better address the desired student knowledge, skills, or abilities.

Curriculum modules will provide AP teachers with the following tools to effectively engage students in the selected topic:

- Enrichment of content knowledge regarding the topic
- Pedagogical content knowledge that corresponds to the topic
- Identification of prerequisite knowledge or skills for the topic
- Explicit connections to AP learning objectives (found in the AP curriculum framework or the course description)
- Cohesive example lessons, including instructional activities, student worksheets or handouts, and/or formative assessments
- Guidance to address student misconceptions about the topic
- Examples of student work and reflections on their performance

The lessons in each module are intended to serve as instructional models, providing a framework that AP teachers can then apply to their own instructional planning.

Note on Internet Resources

All links to online resources were verified at the time of publication. In cases where links are no longer working, we suggest that you try to find the resource by doing a key-word Internet search.

— The College Board

Introduction

This curriculum module presents AP Physics teachers with pedagogy and suggested inquiry activities for introducing students to capacitors and their behavior in circuits. This module consists of four inquiry-based lessons, each of which has several activities. The first lesson is an introduction to electrostatics. The second lesson is about the design and function of parallel-plate capacitors. The third lesson gives students a chance to develop ideas about series and parallel capacitors in circuits. And the fourth lesson looks at the charging and discharging process for capacitors in a circuit with resistance. Together, these four lessons help develop students' understanding of how energy is stored in circuits and how capacitors behave in circuits.

Connections to the Curriculum Framework

This curriculum module builds upon student understanding of electric force, electric field, and potential. Students will extend their understanding of simple circuit models introduced in Physics 1 to include devices that can store separated charge and potential energy. This unit should precede a study of complex circuits with multiple elements (voltage sources, multiple resistors, and capacitors). Note that students will not need to apply Kirchhoff's rules to solve simultaneous equations for circuit quantities, as in Physics C: Electricity and Magnetism.

Simple series and parallel resistor circuits are addressed in the Physics 1 curriculum. The Physics 2 curriculum framework includes capacitors in series and parallel circuits. After completion of the lessons in this module, students should have an understanding of:

- The function of a capacitor
- How capacitance is defined
- How the dimensions and shape of a capacitor determine its capacitance
- How capacitors in series and parallel arrangements behave
- The steady-state behavior of capacitors in circuits containing both resistors and capacitors

The following is a list of the enduring understandings and the associated learning objectives related to capacitor circuits in the Physics 2 curriculum framework. Each learning objective in the curriculum framework is linked with one or more science practices that capture important aspects of the work that scientists engage in. For a list of the AP Science Practices, see Appendix A or the curriculum framework in the *AP Physics 1 and 2 Course and Exam Description*. The science practices enable students to establish lines of evidence and use them to develop and refine testable explanations and predictions of natural phenomena.

Appendix A

Enduring Understandings	Learning Objectives
1.A, 1.B	1.A.5.2, 1.B.1.1, 1.B.1.2, 1.B.2.1, 1.B.2.2, 1.B.2.3
2.C	2.C.2.1
4.E	4.E.3.1, 4.E.3.2, 4.E.3.3, 4.E.3.4, 4.E.3.5, 4.E.4.1, 4.E.4.2, 4.E.5.1, 4.E.5.2, 4.E.5.3
5.B, 5.C	5.B.9.5, 5.B.9.6, 5.C.2.1

Instructional Time and Strategies

This curriculum module consists of four lessons. The first lesson on electrostatics will take approximately one class period, depending on students' level of prior knowledge. This lesson introduces an intriguing device: the indicating electrophorus. Students develop a microscopic model for charge to explain its behavior. You may decide to precede this first activity with a basic investigation of electrostatics, involving frictional charging rods or sticky tape. The other three lessons each comprise two or more activities. You should allow at least one class period (45–50 minutes) for the proper completion of each activity, with the possible exception of Lesson 3, Activity 1, which may take less time.

- Lesson 1: Electrostatics
  - Activity 1: The Indicating Electrophorus

- Lesson 2: Capacitance
  - Activity 1: Introducing the Capacitor
  - Activity 2: Finding a Mathematical Model for Capacitance
  - Activity 3: Extending the Model to Include Energy Storage

- Lesson 3: Capacitor Combinations
  - Activity 1: Predicting Capacitor Combinations
  - Activity 2: Testing Predictions About Capacitor Combinations

- Lesson 4: Capacitors in Circuits
  - Activity 1: Charging and Discharging Capacitor Behavior
  - Activity 2: Collecting Graphs to Describe Capacitor Behavior
  - Activity 3: Experimenting with Capacitors in a Parallel Circuit

The instructional strategies provided throughout this module incorporate a variety of guided inquiry-based activities for students. In several instances, students are introduced to a concept with a brief, engaging demonstration. You do not provide students with explanations for the behavior of the demonstration at this time. After the demonstration, you facilitate as students in lab groups explore the materials themselves by collecting data and making observations along a sequence. You must decide how much guidance to give students; some groups are capable of designing their own labs early in the course, while others will need more guidance at first. After small-group activities, students come together in a whole-class discussion to construct a model for what was observed. After the agreed-upon model is described, students move to other activities to extend the model. You should decide what product to require from students after each lesson. Products may take the form of a writing assignment, a lab report, notes, or a graphic organizer. This inquiry-based instructional approach gives students more autonomy during investigations and they must think about or, if you direct, create essential questions. Students participate in the identification of variables and must predict the behavior of the system. Where possible, allowing students to select aspects of the experimental method causes them to think about what can be measured and why. When students have participated in the process of designing the investigation, their analysis of the data takes on an aspect of "What did we find out?" rather than "Did we get the answer the teacher wants?" This approach, therefore, has the potential to evoke more critical thinking and reasoning about the concepts.

Equation Tables

Equations that students might use in solving problems or answering questions will be provided for them to use during all parts of the AP Physics 2 Exam. It is not intended for students to memorize the equations, so you can feel comfortable in allowing them to use the AP Physics 2 equation tables on all activities and assessments. For the AP Physics 2 equation tables, see Appendix B or the *AP Physics 1 and 2 Course and Exam Description*.

Appendix B

Lesson 1: Electrostatics

Guiding Questions

- What are common properties of conductors and insulators?
- How can an object be charged by frictional charging?
- How can an object be charged by induction?
- What are models of electrostatic interactions?

Lesson Summary

The observation and explanation of the electrophorus can be the basis of the explanation of many electrostatic phenomena including "static shocks," lightning, grain elevator explosions, and gas station explosions. You can also use the Van de Graff generator. Using both an electroscope and an electrophorus, students will explore the processes of frictional charging, charging by induction/polarization, and charging by conduction. By the end of the activity students should be able to make predictions and claims about movement and distribution of charges.

Connections to the Curriculum Framework

The learning objectives aligned to the topic of electrostatics are identified below:

- **Learning Objective (1.B.1.2):** The student is able to make predictions, using the conservation of electric charge, about the sign and relative quantity of net charge of objects or systems after various charging processes, including conservation of charge in simple circuits. [See Science Practices 6.4 and 7.2]
- **Learning Objective (1.B.2.1):** The student is able to construct an explanation of the two-charge model of electric charge based on evidence produced through scientific practices. [See Science Practice 6.2]
- **Learning Objective (1.B.2.2):** The student is able to make a qualitative prediction about the distribution of positive and negative electric charges within neutral systems as they undergo various processes. [See Science Practices 6.4 and 7.2]

- **Learning Objective ([1.B.2.3):** The student is able to challenge claims that polarization of electric charge or separation of charge must result in a net charge on the object. [See Science Practice 6.1]
- **Learning Objective (4.E.3.1):** The student is able to make predictions about the redistribution of charge during charging by friction, conduction, and induction. [See Science Practice 6.4]
- **Learning Objective (4.E.3.2):** The student is able to make predictions about the redistribution of charge caused by the electric field due to other systems, resulting in charged or polarized objects. [See Science Practices 6.4 and 7.2]
- **Learning Objective (4.E.3.3):** The student is able to construct a representation of the distribution of fixed and mobile charge in insulators and conductors. [See Science Practices 1.1, 1.4, and 6.4]
- **Learning Objective (4.E.3.4):** The student is able to construct a representation of the distribution of fixed and mobile charge in insulators and conductors that predicts charge distribution in processes involving induction or conduction. [See Science Practices 1.1, 1.4, and 6.4]
- **Learning Objective (4.E.3.5):** The student is able to plan and/or analyze the results of experiments in which electric charge rearrangement occurs by electrostatic induction, or is able to refine a scientific question relating to such an experiment by identifying anomalies in a data set or procedure. [See Science Practices 3.2, 4.1, 4.2, 5.1, and 5.3]

Student Learning Outcomes

As a result of this lesson, students should be able to:

- Explain the processes of frictional charging, charging by induction/polarization, and charging by conduction
- Given a sequence of events, choose a charge convention (or be given a charge convention, e.g., “assume the friction rod generates a positive charge”) and explain in writing what happened by outlining movement of charge
- Supplement students’ explanation with a “storyboard” with charges drawn on a simple diagram of the events

Student Prerequisite Knowledge

Students are introduced to electric force in AP Physics 1, although the learning objectives are limited and do not include a full treatment of electrostatics. You may want to precede this activity with an activity that will develop the concept of electrostatic attraction, repulsion, and charging by conduction and induction. A “sticky tape” lab (Morse 1992) is a simple, reliable activity that often works even when the humidity is relatively high. In this activity, students pull apart lengths of adhesive tape to produce positively and negatively charged objects. These lengths of tape interact with each other, and with electrically neutral strips of paper and aluminum foil. After making a series of observations, students are given access to a charge of known sign, to determine the signs of their charged tapes. After the sign of each tape is agreed upon, students add plus or minus signs to the diagrams they produced documenting their observations. This activity provides a good lead-in to the more complex and confusing behavior of the indicating electrophorus.

Common Student Misconceptions

Although the basic concepts of electrostatics are often familiar to students from prior study, they may not have been asked to apply the concepts in a logically consistent manner to explain a complex phenomenon. Students typically enter their first physics class with misleading conceptions, such as:

- Electrostatic forces are observed only as charge is transferred.
- Neutral objects do not experience an electrostatic force (no recognition of polarization).
- Both positive and negative charges are transferred.
- Conductors are conduits for charge, but not receptacles.
- Conversely, conductors may be thought of as receptacles for charge, but not conduits.
- Insulators are obstacles to charge.
- Insulators cannot be charged.

In this investigation, these misconceptions will be challenged as students struggle to explain how the indicating electrophorus works.

Teacher Prerequisite Knowledge

You should be familiar with scientific explanations of charging by induction and conduction and with the behavior of the instructional devices frequently used in electrostatics: friction rods, electroscope, electrophorus, and the Van de Graaff generator. Practice with all of the devices used in the lesson is essential; frequently students have trouble reproducing phenomena and you should be able to offer pertinent tips. Familiarity with the sign of charge developed by all combinations of materials under study will be helpful as well. Finally, you will want to represent the charge transfers and charge separations with simple, clear diagrams.

Materials Needed

One electroscope per student group (two to three students)

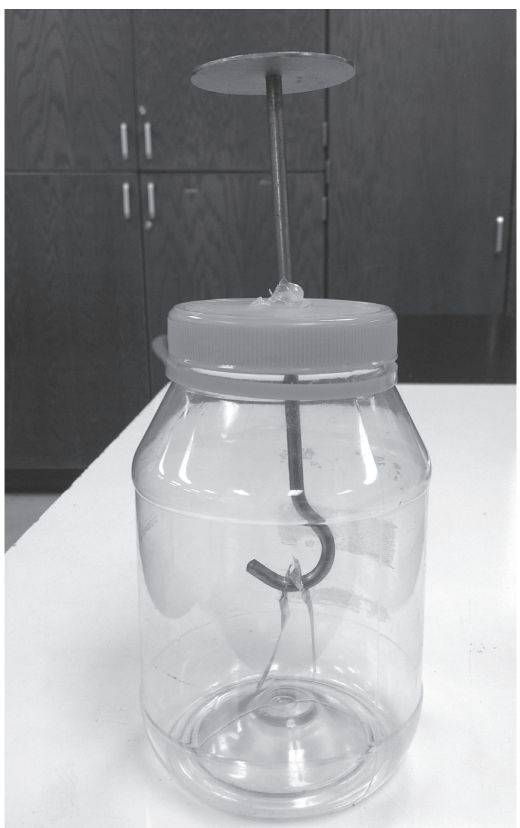
- Mayonnaise jars with lids
- Large paper clips, mass hangers, or stiff, uncoated wire
- Aluminum foil or aluminized Mylar tinsel

One electrophorus per student group (two to three students)

- Aluminum pie pans (one per student group)
- Rigid foam insulation (found in 4′ x 8′ sheets at building supply stores, this insulation is easily cut into pieces of convenient size) or Styrofoam pie plates
- One disposable drinking cup per group
- One coffee stirrer or drinking straw per group
- A portion of an oven-roasting bag per group
- Aluminum foil
- Transparent tape
- String

The electroscope consists of an insulating case (the jar) and a conducting terminal (the wire or paper clip) connected to one or more light-conducting leaves (the aluminum foil). When the terminal is charged or polarized, the leaves respond with movement. If you prepare the electroscopes yourself, allow about an hour's preparation time once you have obtained the materials. (See Figure 1.)

Figure 1



An electrophorus consists of an insulating plate (the foam), a conducting plate (the pie tin), and an insulating handle (the cup). This variant includes a tiny conductor (a small piece of foil) suspended from the handle (by string and straw). The most important design factor is that the foil piece hangs very near to or touching the rim of the aluminum pie pan. (See Figure 2.)

Figure 2

Activity 1: The Indicating Electrophorus

[Time: 45 min]

Review introductory topics of electrostatics that students learned in AP Physics 1 with a simple demonstration. For example, rub an inflated balloon on your own or somebody else's head. The charged balloon will stick to the wall or a ceiling tile for a long time on a dry day. Challenge students to suggest explanations of why the balloon sticks. You may wish to discuss students' explanations right away, or defer them until later. The facts that are essential to explaining the electrophorus include:

1. Charge carriers come in two types.
2. One charge carrier is mobile, the other is not.
3. Like charge carriers produce a repulsive force; unlike charge carriers produce an attractive force.
4. Everyday materials can be classified as either conductors or insulators.
5. Mobile charge carriers remain on insulators unless removed by friction.
6. Conductors contain a proportion of their charge carriers, which are mobile.
7. A neutral insulated conductor can have a uniform charge distribution or be polarized, depending on the presence of other charged objects.
8. An insulated conductor with a net electric charge can have a uniform charge distribution or be polarized, depending on the presence of other charged objects.
9. Charge transfer takes place when there is a net electric force acting on mobile charges in the region of the transfer and ceases when there is no net electric force acting on mobile charges in that region.

Each student group should have their own electrophorus to work with. Students charge the foam by rubbing the oven-roasting bag on it. When the electrophorus is placed on the foam pad, the foil piece will be initially attracted and then repelled. A finger brought near the foil piece will cause it to swing back and forth, transferring charge. If the dangling foil piece is aligned so that it is very near or touching the rim of the pie pan, it may repeatedly swing back and forth on its own, making a distinctive sound as it impacts the pie pan. Challenge students to explain this behavior.

The charged foam is an insulator, and does not readily give up its excess charge to the pie pan. When the pie pan is lifted off the foam and held at a slight distance, nothing happens. If anything, students may note an attraction between the foam and the pie pan, indicating the pie pan has been charged by induction, rather than conduction. The attraction indicates an opposite charge in the charged foam and polarized pie pan. The two would repel each other if a like charge had been conducted from one to the other. When the pie pan is held near the charged foam pad, a spark may jump to a finger held near the top of the pie pan, or the foil piece may begin swinging, even though the pie pan is not placed on the foam pad.

You can show students that it is possible to force a spark from the electrophorus by placing the neutral pie pan on the charged foam pad and briefly touching the top of the pie pan with a finger. When the pie pan is lifted by the handle and brought near a finger again, an audible spark is heard. This process may be repeated many times without much attenuation. But, when the pie pan is placed on the foam pad and not touched, it will not spark when removed from the foam pad. If charge were conducted from the foam pad to the pie pan, it would be possible to produce a spark just by briefly placing the pie pan on the charged foam pad and lifting it up.

Students often do not recognize that the indicator of the electrophorus swings even though charge is not transferred to the aluminum pie pan. Ask students to "Investigate this claim — the electrophorus indicator does not swing because of charge transfer between the foam pad and the pie pan."

Students can use an electroscope to investigate whether charge is transferred or not. When the foam pad touches the electroscope, charge is transferred; but when the aluminum pie pan touches the electroscope, charge is not transferred.

Challenge students to completely explain the behavior of the indicating electrophorus both in words and by drawing diagrams showing the charges on the device as plus and minus signs.

Formative Assessment

One option for formative assessment is to have students create a graphic storyboard: a sequence of diagrams with plus and minus signs indicating excess charge and polarization. The storyboard should illustrate the sequence of events in electrophorus charging and discharging. Consider giving students a starting point ("rubbing the bag on the foam pad"), an ending point ("the foil piece discharges to a nearby finger"), and a number of diagrams to create. This may help prompt students to consider the necessary level of detail. Often their diagrams show charge transfer happening between the foam pad and the pie pan, demonstrating the persistence of the misconception that charge must be transferred from one object to another for electrostatic effects to be observed.

If students' storyboards are inaccurate, you may choose to give them the equivalent of an oral exam. Have students demonstrate the real processes that their diagrams illustrate, while describing to you how the diagrams illustrate what is happening with the materials. When students' diagrams deviate from what is observed, gently prompt them to reconsider. For instance, if students show a charge transfer between the foam pad and the pie pan when there was no opportunity for such a transfer, point out to them that the pie pan will not spark at that point in the process. Ask students to revise their drawings and repeat the oral exam. It may take several tries for students to revise their diagrams until they are accurate. Some students cling tenaciously to the misconception that charge has been transferred if an electrostatic force is observed. If students still fail to explain the electrophorus in terms of induction, you may present a noncontact example. For instance, a charged rod held near an empty soda can on its side may cause the can to roll. When the rod is removed, the rolling stops. Bringing the can near an electroscope shows no charge on it, indicating no charge was transferred. Bringing the rod in close again resumes the rolling. The only explanation is polarization. Mobile charge carriers in the can are displaced in the presence of the charged rod, and an electrostatic force causes the can to roll, even though it has no net charge. Challenging students to draw and explain this example should help.

Lesson 2: Capacitance

Guiding Questions

- How can electric charge be stored?
- How can electrical potential energy be stored?

Lesson Summary

This lesson introduces students to the idea that a device can store charge due to its physical characteristics. The physical quantity is termed *capacitance*. Students initially experiment with homemade parallel-plate capacitors to find how their geometry influences their capacitance. From their investigation, they develop a mathematical model for parallel-plate capacitors. Later, they determine the relationship between potential difference applied to a capacitor and the energy stored inside the capacitor.

Connections to the Curriculum Framework

This lesson addresses the following learning objectives:

- **Learning Objective (1.A.5.2):** The student is able to construct representations of how the properties of a system are determined by the interactions of its constituent substructures. [See Science Practices 1.1, 1.4, and 7 .1]
- **Learning Objective (1.B.1.1):** The student is able to make claims about natural phenomena based on conservation of electric charge. [See Science Practice 6.4]
- **Learning Objective (1.B.1.2):** The student is able to make predictions, using the conservation of electric charge, about the sign and relative quantity of net charge of objects or systems after various charging processes, including conservation of charge in simple circuits. [See Science Practices 6.4 and 7. 2]
- **Learning Objective (1.B.2.3):** The student is able to challenge claims that polarization of electric charge or separation of charge must result in a net charge on the object. [See Science Practice 6.1]

- **Learning Objective (4.E.3.2):** The student is able to make predictions about the redistribution of charge caused by the electric field due to other systems, resulting in charged or polarized objects. [See Science Practices 6.4 and 7.2]
- **Learning Objective (4.E.4.2):** The student is able to design a plan for the collection of data to determine the effect of changing the geometry and/or materials on the resistance or capacitance of a circuit element and relate results to the basic properties of resistors and capacitors. [See Science Practices 4.1 and 4.2]
- **Learning Objective (5.C.2.1):** The student is able to predict electric charges on objects within a system by application of the principle of charge conservation within a system. [See Science Practice 6.4]

Student Learning Outcomes

As a result of this lesson, students should be able to:

- Explain how capacitance of a parallel-plate capacitor depends on its geometry and the dielectric between the plates
- Conduct an experiment to determine the relationship between capacitance and area of a parallel-plate capacitor
- Conduct an experiment to determine the relationship between capacitance and gap between the plates of a parallel-plate capacitor
- Explain how a capacitor stores separated charge and energy
- Predict/calculate the capacitance of a parallel-plate capacitor based on its geometry
- Describe how to arrange conducting plates to construct a capacitor or system of capacitors with a given capacitance

Student Prerequisite Knowledge

Students should have prior experience in electrostatics, including hands-on activities and working with the explanatory model of microscopic charge carriers. Lesson 1 of this module could have provided students with this experience, or you might choose additional electrostatics activities. A web search will turn up many freely available electrostatics labs and demonstrations, often using inexpensive household materials. Some include videos showing the materials in use (see “Franklin and Electrostatics - Ben Franklin as my Lab Partner” in the references). Simulations, while not hands-on, offer a representation of the microscopic behavior of charges. The PhET Interactive Simulations are excellent examples (in particular, Balloons and Static Electricity, Charges and Fields). Each of the PhET simulations includes many free classroom-ready activities.

Students should have already worked with the concepts of charge separation, conduction, induction, and polarization. The nature of conductors and insulators should be established, and it is helpful to have students draw their own representations of what charge is doing in both types of materials during polarization, and when they have a net charge. Students should be aware that there are “free” or “drift” electrons in conductors. Students should be capable of representing electric charge processes with simple diagrams. Electric potential is also important, primarily in Activity 3, where students investigate energy storage in a capacitor. Students will have been introduced to electric potential in AP Physics 1, in the context of circuits.

Important concepts that should precede the lesson are:

- Conservation of energy in a closed system
- Presence of two types of charge carriers
- Conservation of charge in a closed system
- Equivalence of modeling physical situations with either type of charge carrier
- Mobility of charge carriers in conductors
- Relative immobility of charge carriers in insulators
- Electric potential as “electrical potential energy for every one unit of charge”
- Isolated positive charge is at high potential
- An arrangement of charges may store electrical potential energy
- Free positive charge will spontaneously move to low potential

Common Student Misconceptions

Lesson 1 should help establish the nature of conductors and insulators and the effects of polarization. This will counter the student misconception that objects must carry a net charge to experience an electrostatic force. Students also tend to view devices in circuits, including capacitors, as conduits for charge. A capacitor disassembled to show the insulation between its conductors, or a “dissectible” Leyden jar, can be used to show that charge does not pass through a capacitor. Another common misconception is that all of the charge that moves through circuits comes from the battery. Using compass needle deflection as an indicator of charge flow can help counter this. Place compasses under the wires connected to both plates of a capacitor as it charges. Both needles deflect, yet the capacitor contains an insulated gap that prevents charge flowing across it. This demonstrates to students that the moving charge that deflects the needle of the compass connected to the bottom plate has its origin in the wires and the capacitor’s bottom plate, not in the battery.

Teacher Prerequisite Knowledge

This curriculum module is intended to introduce students to capacitance through an inquiry-based instructional approach. You should have practice in leading students in an inquiry-based laboratory format similar to the 5Es instructional model: engagement, exploration, explanation, elaboration, and evaluation. You direct student inquiry by asking guiding questions. The lesson includes some example questions, but you may wish to identify others on your own before beginning the lesson. You should also understand the concepts of electric potential, capacitance, and energy storage.

Activity 1: Introducing the Capacitor

[Time: 45 min]

Materials Needed

- Source of charge, such as friction rods, fur and/or silk, or a Van de Graaff (VDG) generator
- Commercial capacitors for display
- "Dissectible" Leyden jar or pie pans and foam cups
- Neon indicator bulbs
- Disassembled commercial capacitor (optional)

In this activity students investigate the properties of the simplest form of capacitor: the parallel-plate capacitor. To begin, introduce the idea that a device can be made to store charge. Ask students: "In electrostatics, we investigated separating charge for brief periods, but what would it take to store the separated charge? How could you make a device that stores separated charge?" You should be clear that what is meant is a device that stores charge physically, not a battery, which stores energy chemically. You should accept all reasonable answers and lead a brief class discussion of the possibilities. It is not unlikely that some students will know about capacitors, and that other students will find the question intriguing but have little idea how the goal of storing charge could be accomplished. You could ask leading questions about what could be used to keep the charge in place, or what keeps the charge from getting to ground. You should have available a few commercial capacitors and show them as "storage tanks" for separated charge. The interior of many cylinder-form commercial capacitors consists of long, thin layers of conducting material separated by a long, thin layer of insulating material. All of this is tightly rolled to fit inside the capacitor's case. Next, explain to students that the capacity to store charge is called "capacitance," and it is measured in "farads." You should give the definition of capacitance, $C \equiv \frac{Q}{\Delta V}$, and lead students to a verbal articulation of the equation: "the amount of charge a given capacitor can store for every 1 volt of potential difference that is applied to it." A capacitor of 1 farad capacitance stores 1 coulomb of charge when 1 volt of potential difference is applied across it.

At this point you should present a demonstration of capacitance. When charged with a VDG generator, the discharge of a Leyden jar produces a sizeable spark (darken the room to make it more exciting) and an engaging demonstration. The Leyden jar charges best when its outer surface is connected to ground (a faucet works well) and it is elevated so that the discharge electrode is very near the VDG generator. A dissectible version has the advantage that it can be taken apart. It can be taken apart while charged, and when reassembled, it still releases a spark.

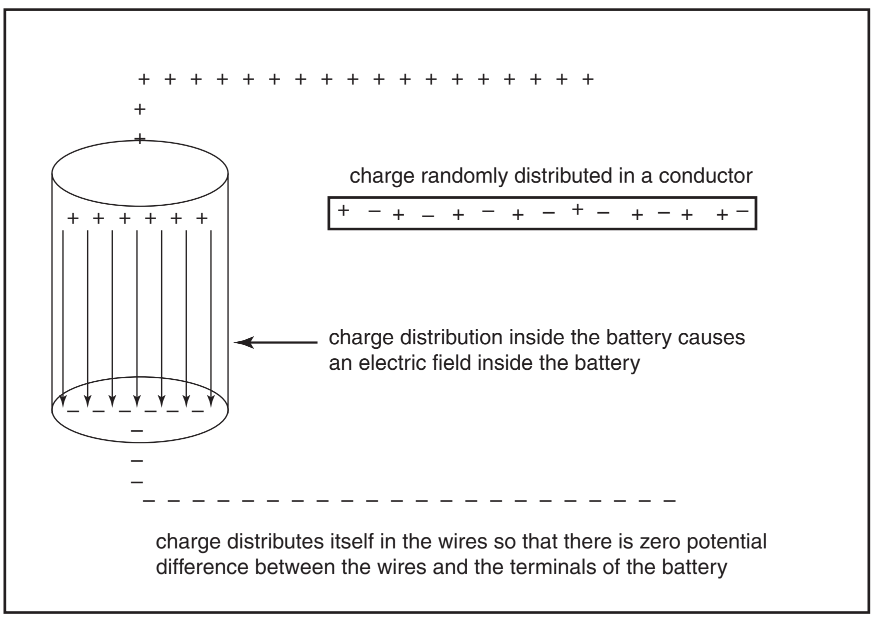
A simple parallel-plate capacitor can be constructed from two aluminum pie pans and two insulating foam cups. Tape the cups to the inside surface of the pie pans. Tape the top of one cup to one of the pie pans, and the bottom of the other cup to the other pie pan so that the capacitor is formed when the two cups are "stacked." This capacitor can be charged by placing an acrylic sheet that has been rubbed with a foam plate on one of the aluminum pans. A neon bulb can be used to demonstrate discharge from this capacitor. The low-potential electrode of the bulb will glow, as charge flows from "high to glow" in the bulb.

Next, you should display the interior structure of a commercial capacitor (thin rolled sheets of alternating metal foil and dielectric). It may be helpful to provide a visual by demonstrating with aluminum foil and plastic sheets. You should pose the question, "On what properties must capacitance depend?" Lead students in a brainstorming session about the different qualities that influence capacitance of the parallel-plate capacitor. Give students a minute to confer with each other, then accept their ideas and write them on the board. Students may suggest variables such as the size of the plates, the potential difference across the capacitor, and the type of insulating material between the plates. Often, students will correctly propose that the capacitance depends on the area of the conducting plates, but they may be incorrect or indeterminate on the relationship of the gap between the plates to the capacitance.

Formative Assessment

For this lesson to progress well, students should have practice in representing charge in different situations. Students could be asked to draw a representation of the charge distribution on a pair of wires or plates connected to a source of potential difference. The electric field of the source induces an uneven distribution of charge, so that the diagram in Figure 3 represents a possible reasonable answer from a student:

Figure 3



In later lessons, you will ask students to represent charge in a capacitor in a similar fashion.

Students are often unsure of how to represent charge distributions. Students may need to be reminded that they are not expected to represent all of the charges present, just enough to represent the model. The goal is for students to have a useful conceptual model for capacitors in mind as they reason through later capacitor problems. When students incorrectly represent a situation, it is important to question them about the implications of what they have shown. If students show charge flow through a capacitor, you could ask them whether the gap is insulating or conductive. Once students recognize that the gap insulates, they should be questioned about why they have indicated charge flow across it. Many students will quickly recognize their mistake and correct it. Another useful technique is for the teacher to copy a student drawing with a "popular" conceptual mistake in it. The teacher presents the drawing to the class as the work of an anonymous student, and asks "What, if anything, is right about this representation?" and "What, if anything, is wrong about this representation?" The class makes a critique of the diagram, highlighting both its good points and weak points.

Activity 2: Finding a Mathematical Model for Capacitance

[Time: 45–60 min]

Materials and Resources Needed

- One or more 4 ft. x 8 ft. sheets of foam insulation with metal foil bonded to each side
- Capacitance meter(s)
- Foam cups
- Aluminum foil
- Handouts 1, 2, 3, and Appendix B

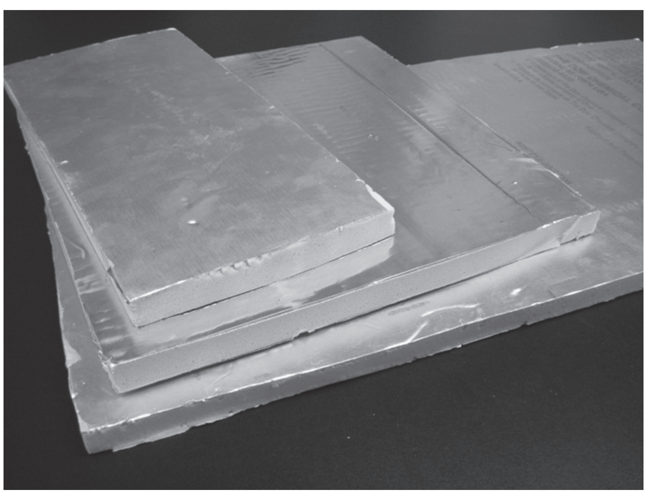
Begin by asking students to brainstorm what qualities of a capacitor could affect its ability to store the most charge with the smallest potential difference across it. Explain that students will be investigating these qualities to produce a conceptual model of capacitance in terms of the dimensions of a set of parallel plates. Students will readily propose the size, although they may not recognize this as the plate area. Students are also likely to suggest material, although they may consider the type of conducting metal of the plates to be more important than the insulating quality of the material between the plates. Students may not bring up the role of the distance between the plates. You may suggest this by asking a leading question about how plates at opposite sides of the room would work compared with plates that are closer together. Students should have come up with a list that includes at least plate size/area and separation distance. You may choose to have students propose investigations of other variables, or you may suggest a narrowing of focus depending on time. This curriculum activity focuses on those geometric quantities that define capacitance.

This lesson calls upon students to make predictions. Students often resist making predictions in unfamiliar contexts. You may wish to provide constraints on the predictions, to make the task clearer and simpler. For instance, you could remind students that this involves predicting whether a particular value, in this case capacitance, would increase, decrease, or remain the same if another variable is changed. After this step, you may encourage students to predict the broad proportionality of the change; in other words, encourage students to think about how the capacitance changes with area. Students could consider whether, when the area doubles, the capacitance doubles or quadruples or decreases by a factor of 2.

The success of an inquiry lesson often depends on your ability to ask questions that cause students to think in the right direction, without giving away the answer. To be successful you should have a good conceptual understanding of the system under study. You should review a complete discussion of how students could possibly know "why" for each aspect of the lesson. Preplanned, overarching ("essential"), and in-the-moment ("Why do you think that?") questions will propel students forward with less frustration and fewer dead ends. It is often helpful for students to identify essential questions on their own, before the lesson begins.

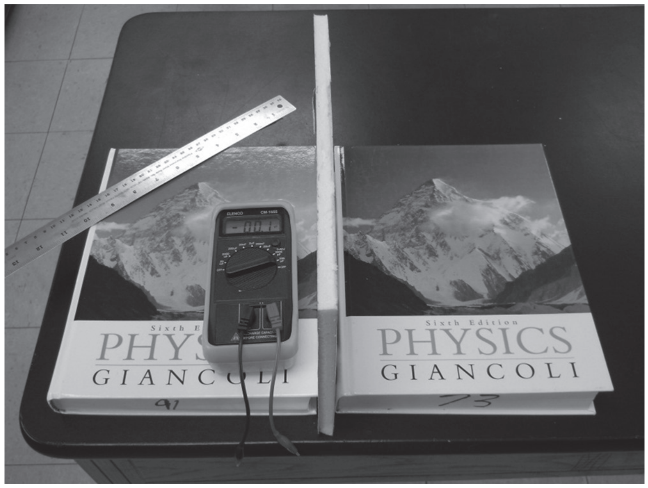
You should explain to students that they will be experimenting with model parallel-plate capacitors. You should prepare the parallel-plate capacitor models from foam insulation material with a metal reflective barrier bonded to each side (this can be purchased inexpensively at building supply stores). These could be cut into regular shapes so that students have a variety of sizes with easy-to-calculate areas (see Figure 4).

Figure 4



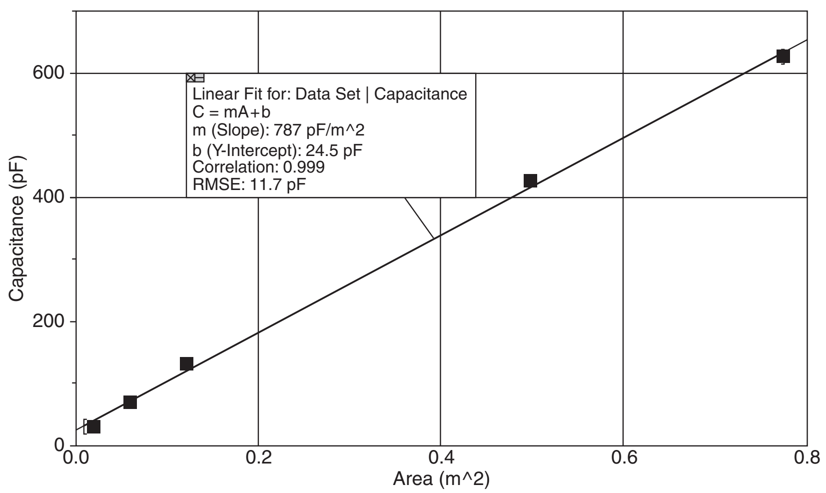
Then you should lead students to carry out the experiment by calculating the area of the capacitors and recording capacitance using the capacitance meter (see Figure 5). The books here are used merely to support the foam capacitor on its edge to make connection to the capacitance meter easier.

Figure 5



A plot of measured capacitance versus area should be linear. Based on this data, students see that parallel-plate capacitance is proportional to plate area. Sample data and graph are shown in Figure 6. The uncertainty of data collected using household materials may be significant. Students should be required to take multiple measurements and complete an uncertainty estimate of their data.

Figure 6



Sample Data

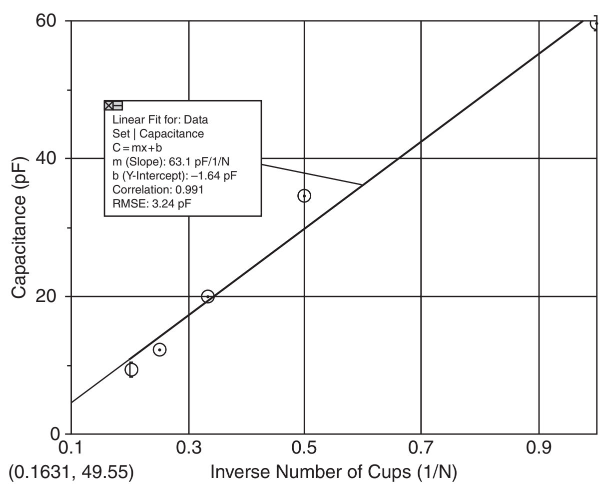
Area (m^2)	Capacitance (pF)
0.02	29.4
0.06	69
0.122	132
0.498	426
0.775	627

The next component of the model is the inverse proportionality of capacitance with distance between the plates. Some science suppliers sell a precisely constructed variable gap capacitor that can be used to determine this relationship from an experiment. After the previous experiment (or at the same time, if possible), students could use this device to perform an experiment to find this relationship. Alternatively, for a lower budget version, have students make "capacitor plates" of aluminum foil, making them very flat and smooth, and insert them into textbooks so that differing numbers of pages intervene. Place a large mass (one or more additional books) on top of the book to press air out of the pages. A plot of capacitance, *C,* versus plate separation, *d* (or number of pages, *N*), yields an inverse relationship. An attempt was made to perform this experiment using plastic transparency sheets, but this did not work. Others have reported similar failures. The same authors report that, for the best results, it is necessary to use solid dielectric material. They suggest Teflon of varying thicknesses.

An alternative version is to sheathe two Styrofoam cups in aluminum foil and insert varying numbers of intervening cups. A plot of *C* versus *N* yields an inverse curve. Students will then plot *C* versus $\frac{1}{N}$,the inverse of the number of cups, to see that capacitance is proportional to the inverse of the number of cups. Students can easily relate this to the gap width (see Figures 7 and 8). Sample data is shown below.

Figure 7

Figure 8



Sample Data

Inverse Number of Cups	Capacitance (pF)
1	59.7
0.5	34.6
0.333	19.9
0.25	12.2
0.2	9.4

Appendix B

After students have determined the relationships between capacitance and area, and between capacitance and plate separation, you can introduce the electric constant, ε_0 (read "epsilon naught"), referred to as the "vacuum permittivity" on the AP Physics 2 Table of Information (Appendix B). Students should understand this as a constant of proportionality that, in this case, relates the geometric properties of the capacitor to the unit of capacitance, the farad. This allows you to present the model for parallel-plate capacitance,

$$C = \kappa\varepsilon_0 \frac{A}{d}$$

where A represents the area of the plates, d the gap between them, and κ (read "kappa") the dielectric constant (a measure of the insulating qualities of the material between the plates). The above activities could be completed in two class periods. For further investigation, students could experiment with different dielectrics between the plates of a capacitor. Some models of variable capacitor from science supply houses come with different dielectric materials (glass, cardboard, and acrylic) to be inserted in between the plates. It is also possible to compare measurements of the capacitance of a pie-plate capacitor with different insulators inserted in between the plates.

Formative Assessment

Handouts 1,2,3

"Ranking tasks" are a class of assignment in which students compare a series of related situations by ranking them from greatest to least or least to greatest as regards some physical quantity. The intent of the task is to guide students to consider the importance of various factors in determining the capacitance of parallel-plate capacitors. The "Capacitance Ranking Task" (Handout 1) in the Handouts section could be used after Activity 2. There are also two "linked multiple-choice tasks" (LMCTs), one on capacitance (Handout 2: "Capacitance"), and the other on storage of charge (Handout 3: "Charge on a Capacitor"). These are also conceptual, but instead of ranking multiple scenarios, students answer a series of multiple-choice questions about a single situation. These questions are conceptual in nature and relate to the effect of changing one or more parameters of the system.

Ranking tasks (or LMCTs) are an excellent stimulus to class discussion. Students initially work individually and silently on the task. When sufficient time has been allowed, students are asked to consult their neighbors, and discuss the physics of one another's answers. They decide on their best answer as a group. You can then take up the task and give feedback individually, or invite one or more groups to present their answer to the class and justify it.

One benefit of using ranking tasks is that on their face, they are simple enough that any student can attempt them. Students' explanations for their rankings reveal their underlying thinking, so they must be encouraged to explain in addition to presenting a ranking. This ranking task can be answered by proportional reasoning from the equation. Students who are not successful may need additional guidance in proportional reasoning. Some students address these tasks intuitively, while others benefit from a clear procedure. To help students who struggle with proportional reasoning, you can present two techniques. One is to set up two algebraic equations with different sets of variables (i.e., A_1, d_1 and A_2, d_2). Suppose one capacitor has double the plate area and double the gap. Students insert the coefficient of variance in the second equation (i.e., change A_2 to $2A_1$ and d_2 to $2d_1$). Substituting into the equation shows students that the two capacitances will be the same.

A second technique is to invent numbers. Students invent numbers that vary in the way specified by the problem, substitute them into the equation, solve, and compare the two answers. Most textbooks include some proportional reasoning tasks in each chapter, and many AP questions use this skill.

Activity 3: Extending the Model to Include Energy Storage

[Time: 45–60 min]

Materials Needed

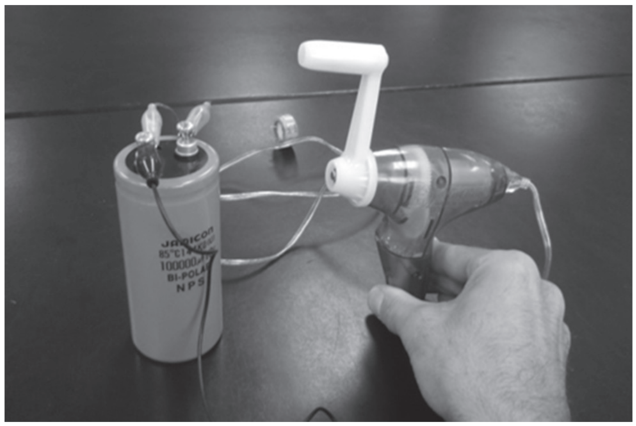
- Hand-cranked electric generator, Genecon or equivalent, one per lab group
- D-cell batteries and holders
- Connecting wires
- For demonstration: “air capacitor” (assembled from household materials, see Figure 9)

This activity depends on having very large capacitors available. The capacitors intended for the CASTLE (Capacitor-Aided System for Teaching and Learning Electricity, distributed by PASCO scientific) curriculum work well (see Figure 9), but other large capacity, low-voltage capacitors will also work. One advantage of capacitors marketed for CASTLE is that they are nonpolar. Students may connect either pole to positive without damage to the capacitors.

Figure 9

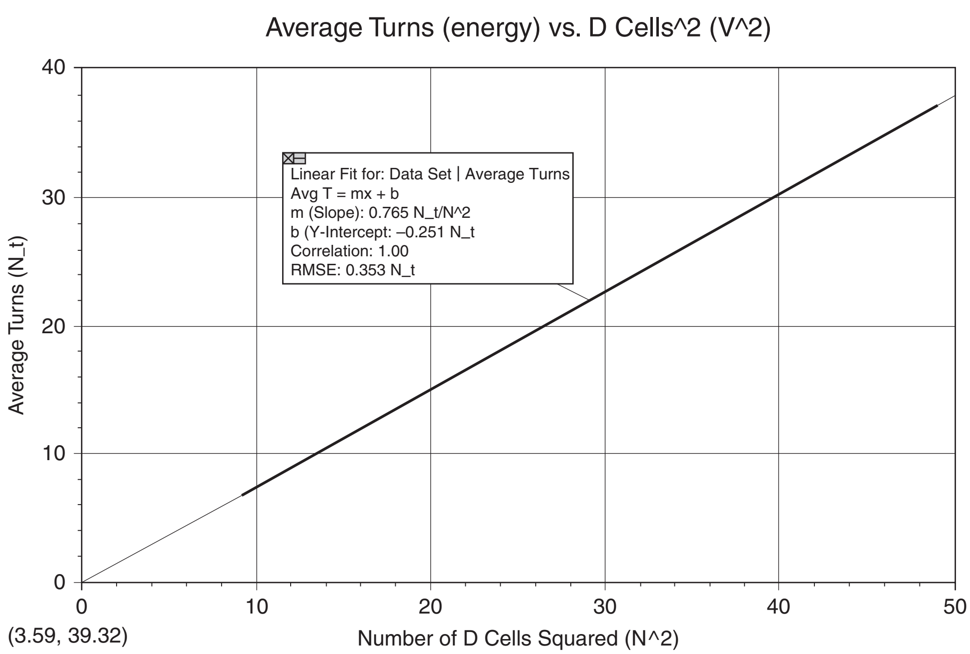
Students conduct this experiment to determine the relationship between potential difference applied to the capacitor and energy stored in the capacitor using a handheld generator. The generator is held with its base on a table, and its connecting leads are connected to a large capacitor (at least 0.025 F) that has been charged with varying numbers of D cells, or a low-voltage power supply at different settings (see Figure 10). The handle of the generator should be free to turn, and the connection with the charged capacitor must be firm from the first instant.

Figure 10



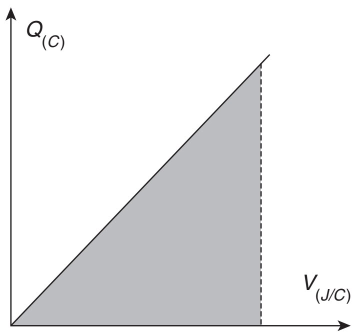
Students count the number of turns the generator handle makes, and plot "Handle Turns" versus "Capacitor Potential Difference" or even "Number of D Cells." Handle turns are proportional to the square of the potential difference, and by inference the energy stored in the capacitor is too. A sample graph of data is shown in Figure 11.

Figure 11



A conceptual "derivation" of the same idea can be shown by examining a graph like the one shown in Figure 12.

Figure 12



As potential difference increases, the charge on the capacitor increases proportionally. The slope represents the capacitance, in units of coulombs per volt. The triangular area is shown, by unit analysis, to be equivalent to energy, since the area of the triangle is $A = \frac{1}{2} b\Delta h$, which has units $\frac{J}{C} \times C = J$. From this, we can infer that $U_C = \frac{1}{2} Q\Delta V$. Since we previously defined capacitance to be $C \equiv \frac{Q}{\Delta V}$, then $Q = C\Delta V$, and by substitution we obtain an alternative equation $U_c = \frac{1}{2} C(\Delta V)^2$ that supports the relationship obtained from the generator activity, since energy is proportional to potential squared. If computer data collection with charge sensors is an option, students can directly measure charge and potential difference, and use these quantities to calculate the energy.

A demonstration "air capacitor" made of water bottles, a balloon, and drinking spouts is a useful model of a capacitor and could be introduced at this time (see Figure 13). Charge in a circuit acts like a "compressible fluid." Charge is everywhere present in the circuit, and flows from excess to deficiency, much like a fluid flows from high to low pressure. The two water bottles model the plates of a capacitor. The balloon models the gap between the plates. The balloon halts the flow of air in the air capacitor, much as the gap between the plates halts the flow of charge across a capacitor. As the balloon inflates, it stores increasing amounts of potential energy. Eventually the balloon's back pressure equals the pressure of the air against it, and the "capacitor" is fully charged, storing energy.

Figure 13

Handout 4

Formative Assessment

In the Handouts section, there is a linked multiple-choice task (Handout 4: "Energy Stored in a Capacitor") about energy storage. Students are presented with a single physical situation and a series of simple changes to that situation. Students are asked to predict how these changes will affect the energy stored in the capacitor. This type of task requires students to think conceptually about the relationships between charge, plate area, plate separation, and so on, and energy storage.

Students have to make several reasoning steps to answer all questions on this task. It is possible to reason from the equation. $U_C = \frac{1}{2}CV^2$ When the capacitance is increased or decreased, a proportional change in energy stored occurs. When the potential is increased, the change in energy stored is proportional to the difference in the squares of the potential, $\Delta U_C \propto \left(V_f^2 - V_i^2\right)$. It is also possible for students to reason conceptually about this relationship. More charge stored implies more energy stored, since energy is transferred as each charge is pushed into place. As the amount of charge increases, the work done to transfer each additional charge to the plates also increases. So there is the effect of adding more charge in addition to the effect of moving the charge to a higher potential." A higher potential difference implies more energy stored, since the definition of potential difference can be summarized as "electrical potential energy per unit of charge."

These tasks could be used as described in the Formative Assessment section for Lesson 1, with students working initially alone, discussing and modifying their work in groups, and receiving feedback directly from the teacher or from a class discussion. If some students are not successful at reasoning about energy using only the equations, one approach is to redirect their attention to the graph shown in Figure 12 of charge (Q) versus potential difference (V). Calculating and comparing the areas related to different potential differences is an additional way to visualize the proportionalities of the energies. And again, with students who struggle with proportional reasoning, it is often helpful to resort to assigning numerical values to the variables and completing a few sample calculations.

Lesson 3: Capacitor Combinations

Guiding Questions

- What happens when capacitors are connected?
- Does connecting capacitors in parallel increase the capacitance?
- Does connecting capacitors in series increase the capacitance?
- Can capacitors be connected so that they share the same potential difference?
- Can capacitors be connected so that they share the same charge?

Lesson Summary

In Activity 1, students make predictions about the ways that capacitors will behave when connected in combination. In Activity 2, students experimentally determine the validity of their predictions.

Connections to the Curriculum Framework

This lesson addresses the following learning objective:.

- **Learning Objective (1.B.1.2):** The student is able to make predictions, using the conservation of electric charge, about the sign and relative quantity of net charge of objects or systems after various charging processes, including conservation of charge in simple circuits. [See Science Practices 6.4 and 7.2]
- **Learning Objective (2.C.2.1):** The student is able to qualitatively and semi-quantitatively apply the vector relationship between the electric field and the net electric charge creating that field. [See Science Practices 2.2 and 6.4]
- **Learning Objective (5.B.9.6):** The student is able to mathematically express the changes in electric potential energy of a loop in a multiloop electrical circuit and justify this expression using the principle of the conservation of energy. [See Science Practices 2.1 and 2.2]

Student Learning Outcomes

As a result of this lesson, students should be able to:

- Gain practice in reasoning about physical systems
- Take a principle learned in one context, and extend it to other contexts
- Use their understanding that capacitance is related to geometry and extend it to parallel capacitors; since plate area is proportional to capacitance, two identical capacitors in parallel have twice the capacitance
- Understand that capacitors in series have less capacitance than a single capacitor alone, because the “electric pressure” is shared between the two series capacitors
- Be able to predict the capacitance of capacitor combinations
- Be capable of designing an experiment to investigate the capacitance of unknown capacitors placed in series or parallel

Student Prerequisite Knowledge

This activity depends in part on an understanding of the concept of potential difference, to which students have been introduced in AP Physics 1. Students should have learned that electric potential difference is a measure of electrical potential energy per unit of charge. Students may find an analogy with gravitational potential helpful. Gravitational potential is defined as energy stored per unit of mass. Each position in the Earth’s gravitational field has a gravitational potential, calculated with reference to a zero position. Activities that may prove helpful include working with topographical contour maps and predicting the amount of gravitational potential. An analogy with a tiny “test mass” may help students visualize a “test charge” when they work on electric fields and electric potential. Another analogy, that potential in a circuit behaves something like “electric pressure,” may also help students visualize what is going on when capacitors are connected in combinations.

Students should have experience with the definition of capacitance (see Lesson 2), and the ability to read and interpret a capacitance meter.

Common Student Misconceptions

Students often think that adding more resistors or more capacitors always increases the associated physical quantity (resistance or capacitance). Potential and potential difference are confusing topics for students. Students may see potential as something that gets “used up” downstream from the source, so that circuit components farther from a battery have less or no potential difference available to them. The idea of electric pressure will prove useful in combating this misconception. Since charge is everywhere present in the circuit components, potential behaves something like pressure in a compressible fluid. It is worth spending some time on this analogy, while simultaneously warning students that, like all analogies, it is not perfect. Charge comes in two types, while there is only one type of molecule in the hypothetical “electric fluid.”

Students may also propose that potential difference is the same for all capacitors in series. This may be a result of confusion with the parallel situation, or an incomplete understanding. Students may think that potential exists in a branch of the circuit as a part of that circuit, without recognizing potential difference as a measure of the energy transferred as charge moves through that part of the circuit. In a similar manner, students may think that charge is different for capacitors of different values in series. The air capacitor analogy should help. If the amount of air displaced in a capacitor depends on the amount of air displaced in an upstream capacitor, then the charge displaced in an electric capacitor is likewise dependent on the charge displaced in an upstream component.

Teacher Prerequisite Knowledge

Before beginning this lesson, you should be familiar with the behavior of capacitors (in terms of charge, potential, and energy) in series and parallel. The ability to make analogies among tanks containing a compressible fluid and capacitors is useful. The air capacitor analogy discussed in Lesson 2 provides a useful "anchor" for student understanding. This conceptual model should provide a basis to support mathematical reasoning.

One goal of this lesson is to derive the equations for capacitors in parallel and series empirically, but you may wish to work out algebraic derivations for the equations, or students may ask questions about the mathematical origins of the equations. Most textbooks contain these derivations, but you may wish to practice them before the lesson begins.

Materials Needed

- Building-material capacitors from previous activity or commercial capacitors
- Textbooks (or other means) to hold building material capacitors upright
- Capacitance meter
- clip leads or strips of aluminum foil
- D-cell batteries, connecting wires, small incandescent bulbs and holders (optional)
- Voltmeters (optional)

Activity 1: Predicting Capacitor Combinations

[Time: 15 min]

You should explain to students that they will now move on to investigations of the behavior of capacitor combinations. Students should be challenged to predict the capacitance, potential, and charge of each capacitor in a parallel and series combination. Since the behavior of capacitors in parallel may be easier to deduce, ask students to predict the capacitance of two or more capacitors in parallel. Since plate area is proportional to capacitance, two identical capacitors in parallel have double plate area and double capacitance. If students are unsure, lead them by asking questions about what happens to the plate area. You might ask students what the potential will be in each capacitor after charge stops moving and reaches equilibrium in the capacitors. Since the plates are connected, ideally the potential will be the same. In an ideal sense, they can be considered the same plate.

Capacitors in series are more difficult for students to predict. When two capacitors are placed in series, the potential difference is shared between the capacitors. If students do not grasp the implications of this, then the following questions may lead them to understand: "What is between the plates of a capacitor?" "If the capacitor is working properly, does charge pass through the insulator?" and "Compare the size of the charge on one plate of a capacitor to the charge on the opposite plate. Trace the charge from the top plate of the first capacitor to the bottom plate of the first capacitor to the top plate of the second capacitor to the bottom plate of the second capacitor. Is there any way the size of the charges could be different?"

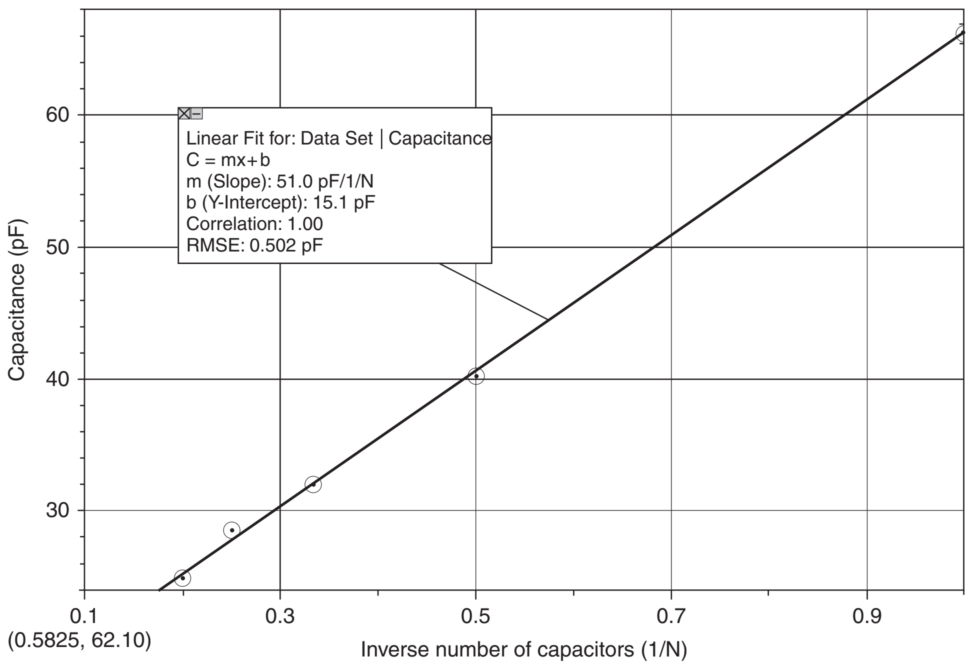
Activity 2: Testing Predictions About Capacitor Combinations

[Time: 45 min]

In this activity, students can work with commercial capacitors, or the building material capacitors from the previous activity. Commercial capacitors can be connected with clip leads. Pieces of aluminum foil, folded flat and taped to homemade capacitors can be used to link them in parallel and series. The capacitance will be measured with the capacitance meter.

If identical capacitors are used, a plot of measured capacitance versus number of capacitors yields a linear best fit with a positive slope for parallel capacitors and an inverse best fit for series capacitors (see Figure 14). You may wish to include one or more nonidentical capacitors to counter possible misconceptions about the graph (i.e., that only integer numbers of capacitors will lie along the best fit). If so, the plot must be total area of plates versus capacitance.

Figure 14



An additional investigation is possible using large capacitors (typically 0.1 *F*, 0.025 *F*, or 1.0 *F*) flashlight bulbs, and a voltage source. Students charge the capacitors and make observations of the behavior of one or more lightbulbs as the capacitor discharges. Students judge the brightness of the bulb and measure the time it is lit to infer charge movement and energy storage in the capacitors. Students can then rank the inferred capacitance of the combinations. Charging the capacitors in series and in parallel can provide evidence for the potential relationship. A series of questions should be directed at students so that they relate the lighting behavior of the bulb to the capacitance. "Compare the brightness and length of time the bulb is lit when the capacitor was arranged in series with the arrangement in parallel. What can you infer about the capacity to store electrical energy in each case?"

You should lead students through a summary "model-building" derivation of the equations for capacitors in series and parallel from their empirical evidence. A graphical derivation from the data may be helpful. For instance, a plot of "measured total capacitance" versus "sum of the capacitances for parallel" should produce a linear best fit with a slope near 1. A plot of "measured total capacitance" versus "sum of the capacitances for series" will produce an inverse curve. If you wish to demonstrate a more rigorous derivation, college-level textbooks have a standard algebraic derivation.

Formative Assessment

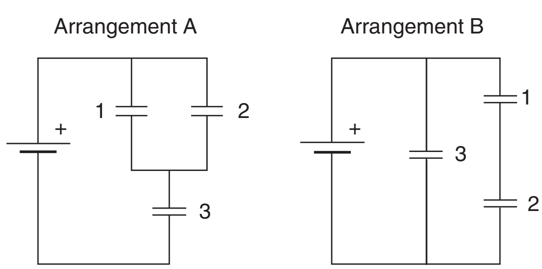
To assess whether students have understood the combination of capacitors, you could assign an opening problem with various combinations of capacitors in parallel and series. A simple example might be to have students calculate the capacitance of a 6-microfarad capacitor and a 3-microfarad capacitor connected in parallel, and then in series. Students could then be asked to write a coherent, paragraph-length explanation about how their answers make sense in terms of the original relationship:

$$C = \kappa\varepsilon_0 \frac{A}{d}$$

Students should see that two parallel capacitors effectively increase the plate area (while keeping the same potential difference) and two series capacitors effectively increases the plate separation (while keeping the same total charge stored).

After students have mastered the relationships for equivalent capacitance of capacitors in simple series and simple parallel arrangements, ask them to extend the models to consider combinations of series and parallel capacitors. For instance, the circuits in Figure 15 contain two possible arrangements of three identical capacitors. When connected to the battery as shown, students might be asked to describe the equivalent capacitance, charge, and potential difference on each of the three numbered capacitors in Arrangement A and Arrangement B. Students could be asked how the two arrangements differ in these three quantities and how can one predict the differences. And, of course, students should be prompted to justify their reasoning.

Figure 15



You could have students check the validity of their predictions by building the actual circuit, and directly measuring the quantities with the capacitance meter and a voltmeter. If a charge sensor is not available, charge on each capacitor may be calculated from the measured capacitance and measured potential difference. Students should be asked to write statements rectifying any incorrect predictions. Their analysis should include a revised justification as well. It is often fruitful to have relatively articulate students present the process to the class, especially if they have had to make substantial revisions to their predictions. Calling on the class to explain the thought process that led to revisions ensures students' accountability.

Students often think that potential is the same everywhere in a circuit. Or, students may think that potential dissipates farther away from the battery. The "compressible fluid" analogy may help students reason about the capacitors in this regard. For instance, in Arrangement A in Figure 15, the "electric pressure" must be distributed equally in Capacitors 1 and 2, because they share a mutual connection.

If students perform poorly on this assessment, have them recall the "air capacitor" demonstration (see Lesson 2, Activity 3) and write about how the "pressure" distributes itself in capacitors joined together. In a series arrangement of air capacitors, there would be a "push back" from one capacitor on the other. In a parallel arrangement, the electric pressure would cause the fluid to flow until the pressure is equalized everywhere. Once any misconceptions have been brought out in the open and discussed, and students have had a chance to modify any incorrect predictions or calculations, you may decide to assess again with a similar but slightly different circuit before proceeding.

Lesson 4: Capacitors in Circuits

Guiding Questions

- How does a capacitor affect other circuit elements when it is in series with them?
- How does a capacitor affect other circuit elements when it is in parallel with them?

Lesson Summary

Students will not be asked to solve quantitative problems involving exponential functions on the AP Physics 2 Exam, but a basic understanding of the charging/discharging behavior of an *RC* circuit is helpful in answering questions about the steady-state behavior of capacitors when connected in a circuit. In this activity, students predict, using conceptual tools of either color coding or drawing charges, the movement of charge when a capacitor is connected in a circuit. In Activity 2, students observe the light produced by a bulb connected to a capacitor during charging and discharging, and compare their observations with their predictions. In (optional) Activity 3, students collect graphs for potential difference across a capacitor to reinforce the results of the previous activities.

Connections to the Curriculum Framework

This lesson addresses the following learning objectives:

- **Learning Objective (1.A.5.2):** The student is able to construct representations of how the properties of a system are determined by the interactions of its constituent substructures. [See Science Practices 1.1, 1.4, and 7 .1]
- **Learning Objective (1.B.1.1):** The student is able to make claims about natural phenomena based on conservation of electric charge. [See Science Practice 6.4]
- **Learning Objective (1.B.1.2):** The student is able to make predictions, using the conservation of electric charge, about the sign and relative quantity of net charge of objects or systems after various charging processes, including conservation of charge in simple circuits. [See Science Practices 6.4 and 7. 2]

- **Learning Objective (1.B.2.3):** The student is able to challenge claims that polarization of electric charge or separation of charge must result in a net charge on the object. [See Science Practice 6.1]
- **Learning Objective (4.E.3.2):** The student is able to make predictions about the redistribution of charge caused by the electric field due to other systems, resulting in charged or polarized objects. [See Science Practices 6.4 and 7.2]
- **Learning Objective (4.E.3.3):** The student is able to construct a representation of the distribution of fixed and mobile charge in insulators and conductors. [See Science Practices 1.1, 1.4, and 6.4]
- **Learning Objective (4.E.3.4):** The student is able to construct a representation of the distribution of fixed and mobile charge in insulators and conductors that predicts charge distribution in processes involving induction or conduction. [See Science Practices 1.1, 1.4, and 6.4]
- **Learning Objective (4.E.4.1):** The student is able to make predictions about the properties of resistors and/or capacitors when placed in a simple circuit, based on the geometry of the circuit element and supported by scientific theories and mathematical relationships. [See Science Practices 2.2 and 6.4]
- **Learning Objective (4.E.5.1):** The student is able to make and justify a quantitative prediction of the effect of a change in values or arrangements of one or two circuit elements on the currents and potential differences in a circuit containing a small number of sources of emf, resistors, capacitors, and switches in series and/or parallel. [See Science Practices 2.2 and 6.4]
- **Learning Objective (4.E.5.2):** The student is able to make and justify a qualitative prediction of the effect of a change in values or arrangements of one or two circuit elements on currents and potential differences in a circuit containing a small number of sources of emf, resistors, capacitors, and switches in series and/or parallel. [See Science Practices 6.1 and 6.4]
- **Learning Objective (4.E.5.3):** The student is able to plan data collection strategies and perform data analysis to examine the values of currents and potential differences in an electric circuit that is modified by changing or rearranging circuit elements, including sources of emf, resistors, and capacitors. [See Science Practices 2.2, 4.2, and 5.1]
- **Learning Objective (5.B.9.5):** The student is able to use conservation of energy principles (Kirchhoff 's loop rule) to describe and make predictions regarding electrical potential difference, charge, and current in steady-state circuits composed of various combinations of resistors and capacitors. [See Science Practice 6.4]

▶ Student Learning Outcomes

As a result of this lesson, students should be able to:

- Describe the observed charging and discharging behavior of capacitors
- Describe the brightness of a lightbulb as the capacitor is in the process of charging and discharging

- Couple the observed brightness of lightbulbs with a "microscopic" explanation of the behavior of the charge inside the capacitor
- Represent with an appropriate diagram the charging or discharging behavior of capacitors

Student Prerequisite Knowledge

Before beginning this lesson, students should be familiar with the idealized behavior of conductors and insulators. A conducting wire is modeled as having zero resistance, and an insulator has infinite resistance. In Physics 1, students should have learned the relationship between potential difference and current in a conductor: Ohm's law ($\Delta V = IR$). Students should review a technique of representing changes in circuits, either by drawing elementary charges as "plus" or "minus" signs, or using a sequence of those drawings to represent changes in a system (i.e., a "storyboard").

Common Student Misconceptions

Student misconceptions about charge are often difficult to change. It does not help that charge is invisible. Simulations that present a microscopic view, such as the PhET simulations, will help students correctly visualize where charge is and what it does under different influences. Particularly prevalent student misconceptions include the following:

- Charge is only present in batteries or other sources.
- Charge that moves through a circuit element must have originated in a battery.
- A circuit element only affects what comes "downstream" from it.

Teacher Prerequisite Knowledge

You should have practice making representations of charge carriers in a circuit. One technique is drawing plus and minus signs on a circuit schematic at different instants, in a storyboard format. You should be prepared to provide students with a conceptual explanation of *RC* circuit behavior. You should be able to point out the subtle differences that distinguish exponential graphs of *RC* circuit charging and discharging. Providing clarifying questions is helpful if students are confused about the differences between the graphs of *RC* circuits and graphs of other functions.

Activity 1: Charging and Discharging Capacitor Behavior

[Time: 45 min]

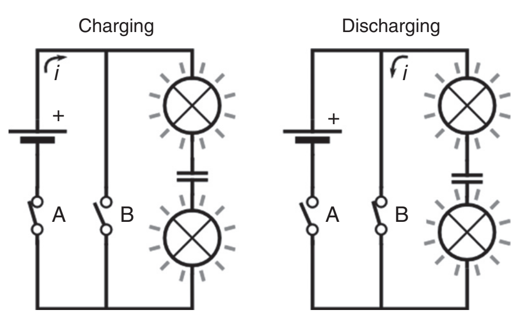
Materials Needed

- Miniature incandescent lightbulbs, such as type 14 or 48 (one or two per group)
- Bulb sockets
- D cells and holders (two to four cells per group) or low-voltage DC power supplies
- Clip leads (two per group)
- CASTLE or other capacitors with very large capacitance 0.025 F or greater (one per group)
- Voltmeters or multimeters (optional, one per group)
- Compasses (one per group)
- Stopwatches (optional)

In this activity, students build a circuit that allows them to observe a very large capacitor charging and discharging, such as the circuit pictured in Figure 16.

To begin the activity, show students a schematic of the circuit and direct them to consider the capacitor initially uncharged. Ask students to predict what happens to the bulbs when the switch is closed and the capacitors charge. Then ask students to predict what happens to the bulbs if both switches are open, the capacitor initially charged, and switch B is closed, allowing the capacitor to discharge. Students' predictions could take the form of a paragraph, a sequence of events, or a series of diagrams showing macroscopic behavior. After making predictions, students build the circuits and test their predictions.

Figure 16



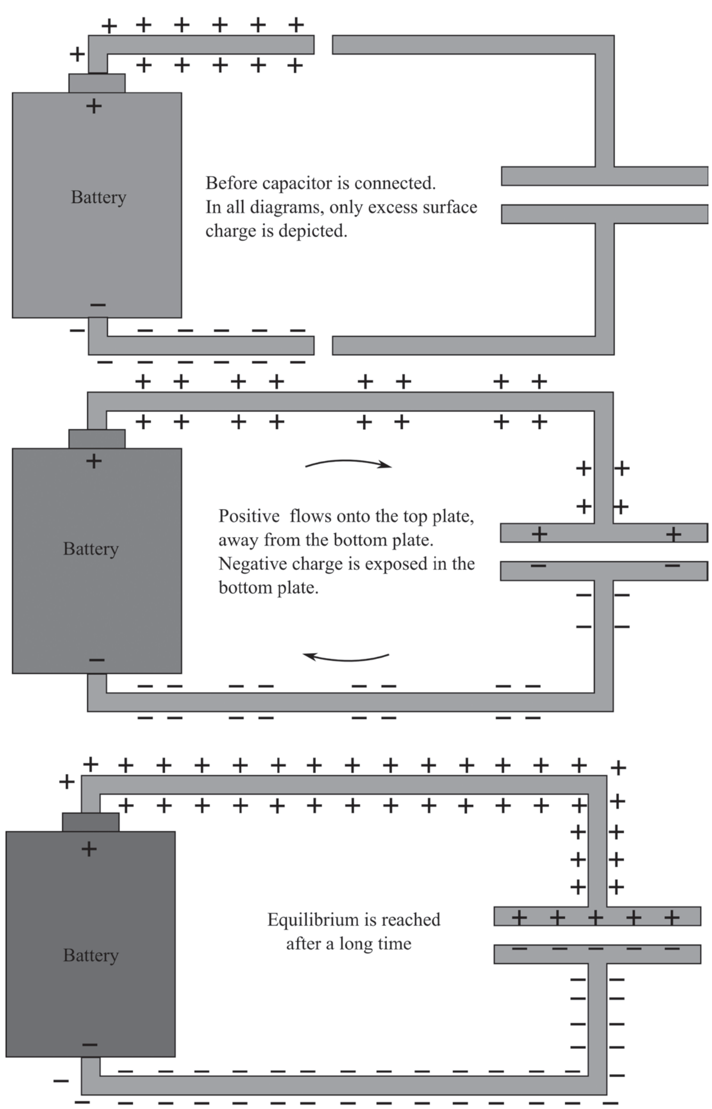
When bulbs are used as indicators of charge flow, it is essential that the capacitors have extremely large capacitance. The miniature "round bulbs" (type 14) are recommended because they have smaller resistance than type 48 "long bulbs," although both types will work. Students are less likely to stop charging before the end of the process because of the smaller charging time when using type 14 bulbs. Students will be able to observe the bulb lighting briefly, a powerful visual confirmation of what is going on in the capacitors.

An additional visual confirmation of the motion of charge occurs when a compass is placed under a wire. Place the compass on the table and have students hold the wire so that it passes directly over the top of the compass, with the wire parallel to the compass needle and directly above it. This will produce the maximum needle deflection for a given current in the wire. In both charging and discharging, the compass deflects strongly at first, but gradually moves back to its initial position. The direction of the deflection is opposite for charging and discharging, indicating that the flow of charge during the charging process is opposite to the flow during discharging.

Formative Assessment

After students have completed their initial observations, you could have them make an annotated diagram explaining what they have observed in Figure 17. It will be helpful to students to review what they have observed/know about capacitor charging before beginning the diagram.

Figure 17



1. The bulbs are bright at first, indicating rapid transfer of energy (and compass needles deflect strongly).
2. The bulbs rapidly grow dimmer (and needle deflection decreases).
3. After a short time, the bulbs dim and go out (compass needle returns to normal orientation).
4. The compass needle near the bottom capacitor plate deflects, indicating charge flows.
5. No charge can flow across the capacitor gap.
6. After a long time, the capacitor has the same potential as the cell(s), and can be disconnected, maintaining its stored energy.

Students should depict with storyboards the process of a capacitor charging (or discharging). The storyboard is a sequence of diagrams that pictures the essential stages of a process. Students may complete the storyboard by using plus/minus signs to represent charge.

You may choose to collect the storyboards and give individual feedback, or direct groups of students to produce whiteboards depicting the process. Student groups may be called upon to present their work to the whole class. You may lead a discussion of the work, highlighting the conceptual successes and leading students to correct flaws. Or, you may direct all student groups to display their work around the room. Students circulate around the room, leaving useful feedback (using dry-erase markers or Post-it Notes) on the whiteboards. After receiving feedback, students should be directed to correct their diagrams until a consensus is reached on a correct representation of the process.

If students still are not visualizing the process correctly, you may direct them to construct a circuit similar to the one under study using a simulation. Observing the behavior of this simulation will help correct any lingering misconceptions.

Activity 2: Collecting Graphs to Describe Capacitor Behavior

[Time: 45 min]

Materials Needed

If computer-based data collection is available:

- Commercial capacitors of any size
- Resistors
- Computer lab interface
- Voltage sensor, current sensor, or charge sensor

If computer-based data collection is not available:

- Very large capacitors
- Resistors
- Multimeter
- Stopwatch

In order to help build conceptual understanding, you may want students to see and collect graphs of charging and discharging behavior as an aid with conceptual understanding. Students should understand that an uncharged capacitor “acts like a wire” when first connected in a circuit and a fully charged capacitor “acts like an open switch” in a circuit.

If computer data collection is available, students can collect current-time data to relate the area of the graphs to the total charge stored, or potential-time data to see the behavior of the “electric pressure” (see Figures 18 and 19). The behavior of the *RC* circuit is associated with the time constant $\tau = RC$, symbolized by the Greek letter tau, and given in units of seconds. A large time constant indicates a large charging/discharging time. After five time constants, nearly all of the change has occurred in an *RC* circuit. Give students a circuit with a time constant large enough, and they can collect data using a voltmeter and a stopwatch.

Figure 18

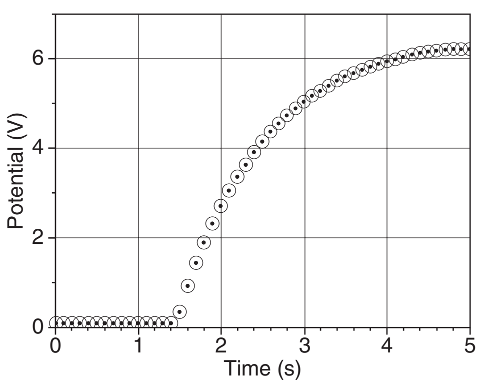
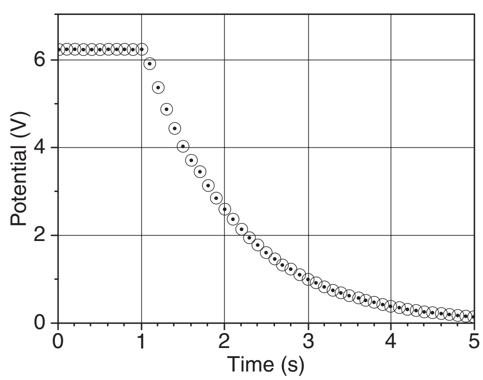


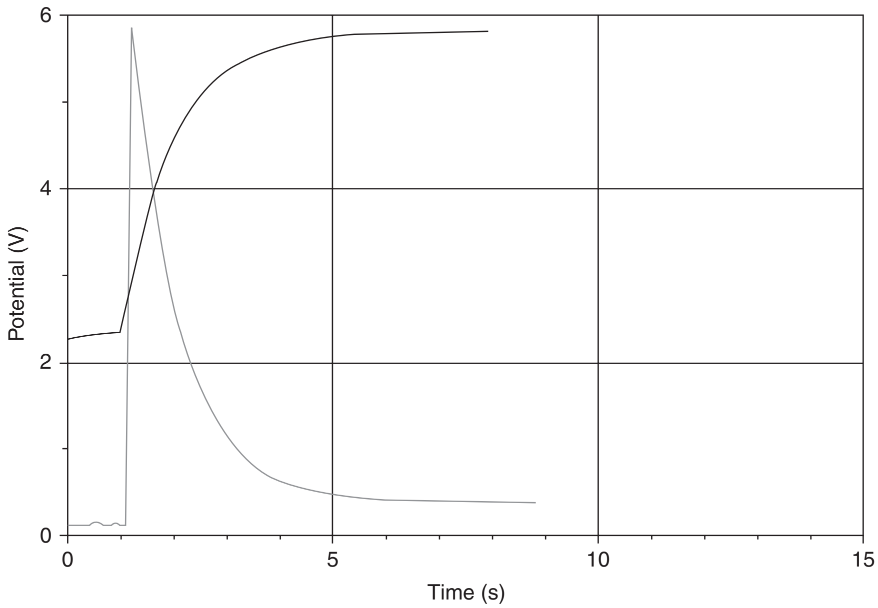
Figure 19



Formative Assessment

Ask students to sketch, directly on a graph of voltage across the capacitor, a graph of voltage across the resistor while the circuit is charging or discharging (see Figure 20). In charging, the potential across the resistor spikes immediately, and then decreases as the capacitor charges. Alternatively, you could give students the graph and ask them to interpret what it shows. Questions may include “Why does the potential difference across the resistor spike so suddenly?” “Why does the charging curve of the resistor look like the ‘mirror image’ of the charging curve of the capacitor?” and “What was the value of the voltage source? Writing a Kirchhoff’s loop rule equation for the *RC* circuit may help: $0 = \varepsilon - \Delta V_C - \Delta V_R$. Initially, the potential drops largely across the resistor, but as the capacitor charges and current decreases, potential drop increases across the capacitor and decreases across the resistor. After a long time, $\varepsilon = \Delta V_C$.”

Figure 20



It is important that students reach some consensus about the behavior of the capacitor that will inform their understanding and allow them to describe the behavior accurately. You may choose to have a class discussion about this formative assessment, and then ask students to self-assess by writing a description of how they performed on the formative assessment, what they now know after the assessment that they did not know before, and what they still need to understand. You should then take the feedback and use it to plan any additional review, or additional practice, if necessary.

Activity 3: Experimenting with Capacitors in a Parallel Circuit

[Time: 45 min]

Materials Needed

- Small incandescent lightbulbs (such as type 14 or 48)
- D cells and holders, or low-voltage DC power supplies
- Connecting wires
- Capacitors with very large capacitance (0.025 F or greater)
- Voltmeters or multimeters (optional)

Show students one or both of the circuit diagrams (see Figures 21 and 22), and tell them that they will build the circuit(s) with large capacitors.

Figure 21

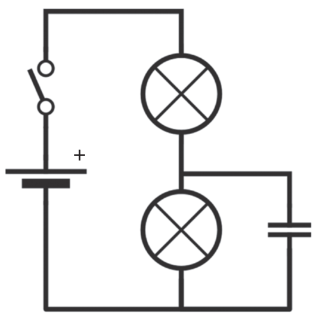
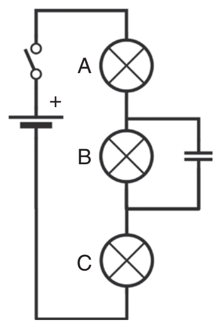


Figure 22



Have students write down a prediction for the behavior of all the bulbs and the capacitor (initially uncharged) when the switch is closed. Using their physical circuit students should observe what happens when the switch is closed with the capacitor initially discharged. After the switch is closed, Bulb 1 (and Bulb 3 in Figure 22) lights brightly and Bulb 2 is initially unlit. As the capacitor is charged, Bulb 2 becomes brighter and brighter. After steady state is reached, all the series bulbs will be identical in brightness, assuming their resistances are identical. Students should be able to explain what happens using their earlier observations of the capacitor.

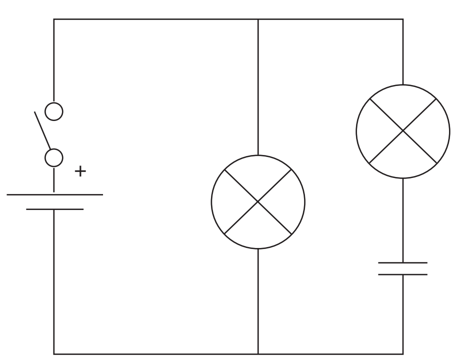
After this observation is complete, students should open the switch with the capacitor fully charged. Challenge students to explain what they have observed. Bulb 2 briefly lights as charge flows in the opposite direction as before. Students could also be asked to storyboard this situation, showing different instants of time as charge builds up on the capacitors until they are fully charged.

Formative Assessment

The circuits above contain capacitors in parallel with a bulb. As a formative assessment, ask students to predict the behavior of a circuit with bulbs in parallel, with one or more of the bulbs in series with a capacitor.

For example, have students consider the circuit in Figure 23:

Figure 23



Then ask students to do the following:

- Describe the changes that occur in each bulb at the instant the switch is closed until the circuit reaches a steady-state condition. In students' descriptions, they should include both the directly observable changes and the measurable changes in physical quantities like potential difference, current, and energy. Students may use diagrams, calculations, or graphs to support their answers.
- After the circuit reaches steady state, the switch is opened again. Describe the changes that occur in each bulb at the instant the switch is opened until the circuit reaches a steady-state condition again. In students' descriptions, they should include both the directly observable changes and the measurable changes in physical quantities like potential difference, current, and energy. Students may use diagrams, calculations, or graphs to support their answers.

You should provide feedback directly to students at this point. Students may perform poorly at this task because of a failure to connect the graphs/concepts from the activity. You might direct these students to repeat this task for a different circuit, after making a list of the concepts that describe the behavior of a capacitor: when the switch is initially closed the capacitor offers no resistance to charge flow, after a long time the capacitor gains an "electric pressure" equivalent to that of the battery, and so on.

For students who continue to struggle, it is helpful to have them connect capacitors in the circuit and explain the actual behavior. The PhET simulation "Circuit Construction Kit: (AC + DC)" (see Reference below) has capacitors that students can connect in the circuits shown above, with bulbs. Students can then use the simulation to visualize the motion electrons (and infer the flow of conventional current) as the switch is opened and closed. Students should be directed in particular to observe the initial moment just after the switch is moved, and also the steady state achieved a "long" time after the switch has been moved.

Summative Assessment

Several types of summative assessment could follow this sequence of instruction. AP Physics 2 emphasizes depth of understanding. One method of assessing deeper understanding is to ask students to write or produce new representations (e.g., sketch a graph for two physical quantities). Modifying an existing AP or textbook problem to cause students to write more, explain more, or produce new representations is an option for summative assessment of this curriculum module.

An open-ended writing assignment that asks students to describe or perform some tasks similar to what they have done in the module will help to expose student misunderstandings and highlight student understandings. Students could be asked to design an experiment to investigate one or more physical properties of capacitors, describe the behavior of a circuit or circuits containing capacitors, or explain to a person who is not knowledgeable about physics what a capacitor is and how it behaves. Students should be encouraged to incorporate multiple representations into their product: sample graphs, circuit diagrams, and representations of the microscopic behavior of charge would all be appropriate.

A lab practical assesses student understanding as well as ability to manipulate lab materials. Students could be given a set of materials, asked to design a lab to investigate a relationship, predict the relationship, and then carry out the procedure. Data and analysis would complete the lab practical assessment. A problem-based lab practical is also a possibility. Students could be asked to create a circuit that performs a specific task. For instance, “Given three bulbs, two capacitors, connecting wires, batteries, and a switch, create a circuit in which all three bulbs glow but one goes out when the switch is closed.”

References

American Modeling Teachers Association. Accessed November 11, 2013. http://modelinginstruction.org.

BSCS 5E Instructional Model. Accessed November 11, 2013. www.bscs.org/bscs-5e-instructional-model.

Grove, T. T., & Masters, M. F. (2009). Simply and accurately measuring dielectric constants using a parallel plate capacitor: Sometimes you just can't win. *The Physics Teacher*, *47*, 312 .

Grove, T. T., Masters, M. F., & Miers, R. E. (2005). Determining dielectric constants using a parallel plate capacitor. *American Journal of Physics*, *73*, 52–56.

Hieggelke, C. J., Maloney, D. P., O'Kuma, T. L., & Kanim, S. E. (2006). *E&M TIPERs: Electricity & magnetism tasks inspired by physics education research*. Upper Saddle River, NJ: Prentice Hall.

Kraftmakher, Y. (2012). Charge and energy stored in a capacitor. *The Physics Teacher*, *50*, 73.

Morse, R. A. *Franklin and Electrostatics- Benjamin Franklin as my Lab Partner (2004)*. Accessed March 14, 2014. www.compadre.org/psrc/franklin.

Morse, R. (1992). *Teaching about electrostatics: An AAPT/PTRA workshop manual*. College Park, MD: American Association of Physics Teachers.

O'Kuma, T. L., Maloney, D. P., & Hieggelke, C. J. (2008). *Ranking task exercises in physics*. Upper Saddle River, NJ: Prentice Hall.

PASCO: CASTLE Kit–EM-8624A. Accessed July 2, 2012. www.pasco.com/prodCatalog/EM/EM-8624_castle-kit/index.cfm#overviewTab.

PhET Balloons and Static Electricity. Accessed February 9, 2014. phet.colorado.edu/en/simulation/balloons.

PhET Charges and Fields. Accessed February 9, 2014. phet.colorado.edu/en/simulation/charges-and-fields.

PhET Circuit Construction Kit (AC + DC). Accessed November 14, 2013. phet.colorado.edu/en/simulation/circuit-construction-kit-ac.

Steinberg, M. and Wainwright, C. Using Models to Teach Electricity – The CASTLE project. The Physics Teacher (Volume 31, Issue 6, Pages 353-357)

Young, D. (2006). Exploring series and parallel combinations of capacitors by inquiry. *The Physics Teacher*, *44*, 366–368.

Resources

This textbook does a good job of presenting concepts along with mathematics:

Knight, R. D., Jones, B., & Field, S. (2012). *College physics: A strategic approach*. (2nd ed.). Boston: Addison Wesley (Pearson Education).

The associated workbook contains many conceptual tasks that are useful for formative assessment:

Knight, R. D., & Andrews, J. H. (2010). *College physics: A strategic approach* [student workbook]. Boston: Addison Wesley (Pearson Education).

Extremely well-designed simulation of capacitors that allows students to "see inside" capacitors to visualize charge accumulating on the plates:

PhET Simulations—"Capacitor Lab Circuit Construction Kit (AC + DC)." Accessed October 18, 2013. phet.colorado.edu.

An excellent resource for inquiry lessons in physics, the Rutgers PAER website contains learning cycle-based lessons with videos to develop many of the major concepts in introductory physics through inquiry:

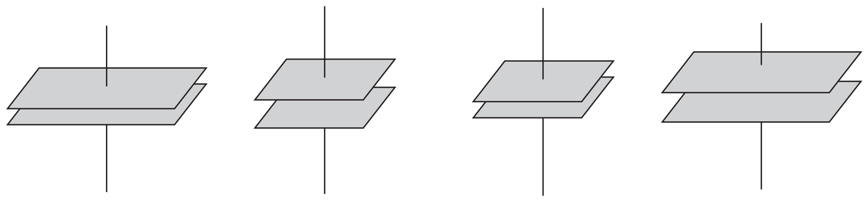
Rutgers Learning Cycles on Electricity and Magnetism. Accessed March 14, 2014. paer.rutgers.edu/pt3/cycleindex.php?topicid=10.

Handout 1

Capacitance Ranking Task

Consider the four capacitors shown below. The plate areas and plate separations are as shown in the diagrams. Each capacitor has air as the dielectric between the plates.

Rank the capacitors from greatest to least based on their capacitance.



Capacitor A
Area = 2A
Separation = D

Capacitor B
Area = A
Separation = 2D

Capacitor C
Area = A
Separation = D

Capacitor D
Area = 2A
Separation = 2D

(Greatest) 1 ________________ 2 ________________ 3 ________________ 4 ________________ (Least)

Or, all of them have the same capacitance __________

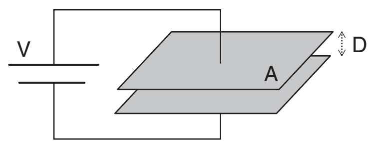
Carefully explain your reasoning.

Handout 2

Capacitance

Linked Multiple-Choice Task

Two parallel plates of area *A* are separated by a distance *D*. They are connected to a battery of voltage ΔV . When the battery is connected, the charge on the capacitor is *Q*. A few changes are made to this arrangement, as described below. For each individual change, indicate what effect this change will have on the **capacitance** of the arrangement.



Use the following answers for your choices:

I — This will increase the capacitance of the arrangement

D — This will decrease the capacitance of the arrangement

NC — There will be no changes to the capacitance of the arrangement

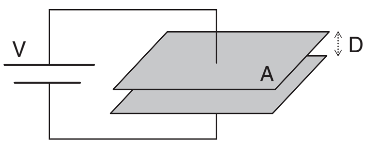
1. The plate area is increased to 2*A*. ________________
2. The plate separation is increased to 2*D*. ________________
3. The battery voltage is increased to 2*V*. ________________
4. The plate area is decreased to $\frac{A}{2}$. ________________
5. The battery voltage is decreased to $\frac{V}{2}$. ________________
6. The plate area is increased to 2*A* and the plate separation is increased to 2*D*. ________________
7. The charge on the plates is increased to 2*Q*. ________________
8. The plate area is decreased to $\frac{A}{2}$ and the plate separation is increased to 2*D*. ________________

Handout 3

Charge on a Capacitor

Linked Multiple-Choice Task

A parallel-plate capacitor has plate area A and plate separation D. It is connected to a battery of voltage ΔV. When the battery is connected, the charge on the capacitor is Q. A few changes are made to this arrangement, as described below. For each individual change, indicate what effect this change will have on the **charge stored** on the capacitor.



Use the following answers for your choices:

I — This will increase the charge stored on the capacitor

D — This will decrease the charge stored on the capacitor

NC — There will be no change to the charge stored on the capacitor

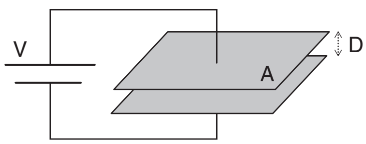
1. The plate area is increased to $2A$. ____________
2. The plate separation is increased to $2D$. ____________
3. The battery voltage is increased to $2V$. ____________
4. The plate area is decreased to $\frac{A}{2}$. ____________
5. The battery voltage is decreased to $\frac{V}{2}$. ____________
6. The plate area is increased to $2A$ and the plate separation is increased to $2D$. ____________
7. The battery voltage is decreased to $\frac{V}{2}$ and the plate separation is decreased to $\frac{D}{2}$. ____________
8. The plate area is decreased to $\frac{A}{2}$ and the plate separation is increased to $2D$. ____________

Handout 4

Energy Stored in a Capacitor

Linked Multiple-Choice Task

A parallel-plate capacitor has plate area *A* and plate separation *D*. It is connected to a battery of voltage ΔV . When the battery is connected, the charge on the capacitor is *Q*. A few changes are made to this arrangement, as described below. For each individual change, indicate what effect this change will have on the **energy stored** in the capacitor.



Use the following answers for your choices:

I — This will increase the energy stored in the capacitor

D — This will decrease the energy stored in the capacitor

NC — There will be no change to the charge stored on the capacitor

1. The plate area is increased to 2A. ________________
2. The plate separation is increased to 2D. ________________
3. The battery voltage is increased to 2V. ________________
4. The plate area is decreased to $\frac{A}{2}$. ________________
5. The battery voltage is decreased to $\frac{V}{2}$. ________________
6. The plate area is increased to 2A and the separation is increased to 2D. ________________
7. The battery voltage is decreased to $\frac{V}{2}$ and the plate separation is decreased to $\frac{D}{2}$. ________________
8. The plate area is decreased to $\frac{A}{2}$ and the plate separation is increased to 2*D*. ________________

Appendix A

Science Practices for AP Courses

Science Practice 1: The student can use representations and models to communicate scientific phenomena and solve scientific problems.

- 1.1: The student can *create representations and models* of natural or man-made phenomena and systems in the domain.
- 1.2: The student can *describe representations and models* of natural or man-made phenomena and systems in the domain.
- 1.3: The student can *refine representations and models* of natural or man-made phenomena and systems in the domain.
- 1.4: The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.
- 1.5: The student can *reexpress key elements* of natural phenomena across multiple representations in the domain.

Science Practice 2: The student can use mathematics appropriately.

- 2.1: The student can *justify the selection of a mathematical routine* to solve problems.
- 2.2: The student can *apply mathematical routines* to quantities that describe natural phenomena.
- 2.3: The student can *estimate numerically* quantities that describe natural phenomena.

Science Practice 3: The student can engage in scientific questioning to extend thinking or to guide investigations within the context of the AP course.

- 3.1: The student can *pose scientific questions*.
- 3.2: The student can *refine scientific questions*.
- 3.3: The student can *evaluate scientific questions*.

Science Practice 4: The student can plan and implement data collection strategies appropriate to a particular scientific question.

- 4.1: The student can *justify the selection of the kind of data* needed to answer a particular scientific question.
- 4.2: The student can *design a plan* for collecting data to answer a particular scientific question.
- 4.3: The student can *collect data* to answer a particular scientific question.
- 4.4: The student can *evaluate sources of data* to answer a particular scientific question.

Science Practice 5: The student can perform data analysis and evaluation of evidence.

- 5.1: The student can *analyze data* to identify patterns or relationships.
- 5.2: The student can *refine observations and measurements* based on data analysis.
- 5.3: The student can *evaluate the evidence provided by data sets* in relation to a particular scientific question.

Science Practice 6: The student can work with scientific explanations and theories.

- 6.1: The student can *justify claims with evidence*.
- 6.2: The student can *construct explanations of phenomena based on evidence* produced through scientific practices.
- 6.3: The student can *articulate the reasons that scientific explanations and theories are refined or replaced*.
- 6.4: The student can *make claims and predictions about natural phenomena* based on scientific theories and models.
- 6.5: The student can *evaluate alternative scientific explanations*.

Science Practice 7: The student is able to connect and relate knowledge across various scales, concepts, and representations in and across domains.

- 7.1: The student can *connect phenomena and models* across spatial and temporal scales.
- 7.2: The student can *connect concepts* in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

Appendix B

Table of Information and Equation Tables for AP Physics 2

ADVANCED PLACEMENT PHYSICS 2 EQUATIONS, EFFECTIVE 2015

CONSTANTS AND CONVERSION FACTORS

Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1 \text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^8$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23}$ mol^{-1}	Universal gravitational constant, $G = 6.67 \times 10^{-11}$ m^3/kg•s^2
Universal gas constant, $R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface, $g = 9.8$ m/s^2
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	

1 unified atomic mass unit,	$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34}$ J•s $= 4.14 \times 10^{-15}$ eV•s
	$hc = 1.99 \times 10^{-25}$ J•m $= 1.24 \times 10^3$ eV•nm
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/N•m^2
Coulomb's law constant,	$k = 1/4\pi\varepsilon_0 = 9.0 \times 10^9$ N•m^2/C^2
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}$ (T•m)/A
Magnetic constant,	$k' = \mu_0/4\pi = 1 \times 10^{-7}$ (T•m)/A
1 atmosphere pressure,	$1 \text{ atm} = 1.0 \times 10^5 \text{ N/m}^2 = 1.0 \times 10^5$ Pa

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES

Factor	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following conventions are used in this exam.

I. The frame of reference of any problem is assumed to be inertial unless otherwise stated.
II. In all situations, positive work is defined as work done <u>on</u> a system.
III. The direction of current is conventional current: the direction in which positive charge would drift.
IV. Assume all batteries and meters are ideal unless otherwise stated.
V. Assume edge effects for the electric field of a parallel plate capacitor unless otherwise stated.
VI. For any isolated electrically charged object, the electric potential is defined as zero at infinite distance from the charged object.

ADVANCED PLACEMENT PHYSICS 2 EQUATIONS, EFFECTIVE 2015

MECHANICS

$$v_x = v_{x0} + a_x t$$

$$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{x0}^2 + 2a_x (x - x_0)$$

$$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$$

$$|\vec{F}_f| \le \mu |\vec{F}_n|$$

$$a_c = \frac{v^2}{r}$$

$$\vec{p} = m\vec{v}$$

$$\Delta \vec{p} = \vec{F}\,\Delta t$$

$$K = \frac{1}{2} m v^2$$

$$\Delta E = W = F_{\parallel} d = F d \cos\theta$$

$$P = \frac{\Delta E}{\Delta t}$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$x = A\cos(\omega t) = A\cos(2\pi f t)$$

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$$

$$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$$

$$\tau = r_{\perp} F = r F \sin\theta$$

$$L = I\omega$$

$$\Delta L = \tau\,\Delta t$$

$$K = \frac{1}{2} I \omega^2$$

$$|\vec{F}_s| = k|\vec{x}|$$

$$U_s = \frac{1}{2} k x^2$$

$$\Delta U_g = m g\,\Delta y$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

$$T_p = 2\pi\sqrt{\frac{\ell}{g}}$$

$$|\vec{F}_g| = G\frac{m_1 m_2}{r^2}$$

$$\vec{g} = \frac{\vec{F}_g}{m}$$

$$U_G = -\frac{G m_1 m_2}{r}$$

a = acceleration
A = amplitude
d = distance
E = energy
F = force
f = frequency
I = rotational inertia
K = kinetic energy
k = spring constant
L = angular momentum
ℓ = length
m = mass
P = power
p = momentum
r = radius or separation
T = period
t = time
U = potential energy
v = speed
W = work done on a system
x = position
y = height
α = angular acceleration
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed

ELECTRICITY AND MAGNETISM

$$|\vec{F}_E| = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}$$

$$\vec{E} = \frac{\vec{F}_E}{q}$$

$$|\vec{E}| = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}$$

$$\Delta U_E = q\Delta V$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$|\vec{E}| = \left|\frac{\Delta V}{\Delta r}\right|$$

$$\Delta V = \frac{Q}{C}$$

$$C = \kappa\varepsilon_0 \frac{A}{d}$$

$$E = \frac{Q}{\varepsilon_0 A}$$

$$U_C = \frac{1}{2} Q\Delta V = \frac{1}{2} C(\Delta V)^2$$

$$I = \frac{\Delta Q}{\Delta t}$$

$$R = \frac{\rho\ell}{A}$$

$$P = I\,\Delta V$$

$$I = \frac{\Delta V}{R}$$

$$R_s = \sum_i R_i$$

$$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$$

$$C_p = \sum_i C_i$$

$$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$$

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
ℓ = length
P = power
Q = charge
q = point charge
R = resistance
r = separation
t = time
U = potential (stored) energy
V = electric potential
v = speed
κ = dielectric constant
ρ = resistivity
θ = angle
Φ = flux

$$\vec{F}_M = q\vec{v} \times \vec{B}$$

$$|\vec{F}_M| = |q\vec{v}||\sin\theta||\vec{B}|$$

$$\vec{F}_M = I\vec{\ell} \times \vec{B}$$

$$|\vec{F}_M| = |I\vec{\ell}||\sin\theta||\vec{B}|$$

$$\Phi_B = \vec{B}\bullet\vec{A}$$

$$\Phi_B = |\vec{B}|\cos\theta|\vec{A}|$$

$$\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t}$$

$$\mathcal{E} = B\ell v$$

ADVANCED PLACEMENT PHYSICS 2 EQUATIONS, EFFECTIVE 2015

FLUID MECHANICS AND THERMAL PHYSICS

$$\rho = \frac{m}{V}$$

$$P = \frac{F}{A}$$

$$P = P_0 + \rho g h$$

$$F_b = \rho V g$$

$$A_1 v_1 = A_2 v_2$$

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$\frac{Q}{\Delta t} = \frac{kA\,\Delta T}{L}$$

$$PV = nRT = Nk_B T$$

$$K = \frac{3}{2}k_B T$$

$$W = -P\,\Delta V$$

$$\Delta U = Q + W$$

A = area
F = force
h = depth
k = thermal conductivity
K = kinetic energy
L = thickness
m = mass
n = number of moles
N = number of molecules
P = pressure
Q = energy transferred to a system by heating
T = temperature
t = time
U = internal energy
V = volume
v = speed
W = work done on a system
y = height
ρ = density

MODERN PHYSICS

$$E = hf$$

$$K_{\max} = hf - \phi$$

$$\lambda = \frac{h}{p}$$

$$E = mc^2$$

E = energy
f = frequency
K = kinetic energy
m = mass
p = momentum
λ = wavelength
ϕ = work function

WAVES AND OPTICS

$$\lambda = \frac{v}{f}$$

$$n = \frac{c}{v}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f}$$

$$|M| = \left|\frac{h_i}{h_o}\right| = \left|\frac{s_i}{s_o}\right|$$

$$\Delta L = m\lambda$$

$$d\sin\theta = m\lambda$$

d = separation
f = frequency or focal length
h = height
L = distance
M = magnification
m = an integer
n = index of refraction
s = distance
v = speed
λ = wavelength
θ = angle

GEOMETRY AND TRIGONOMETRY

Rectangle
$A = bh$

Triangle
$A = \frac{1}{2}bh$

Circle
$A = \pi r^2$
$C = 2\pi r$

Rectangular solid
$V = \ell w h$

Cylinder
$V = \pi r^2 \ell$
$S = 2\pi r \ell + 2\pi r^2$

Sphere
$V = \frac{4}{3}\pi r^3$
$S = 4\pi r^2$

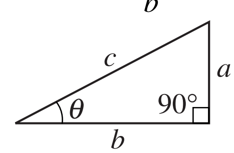
A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius

Right triangle
$c^2 = a^2 + b^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$



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