AP CALCULUS AB/BC

Scoring Guidelines

Part A (AB or BC): Graphing Calculator Required

t (hours)	0	2	4	6	8	10	12
$R(t)$ (vehicles per hour)	2935	3653	3442	3010	3604	1986	2201

1. On a certain weekday, the rate at which vehicles cross a bridge is modeled by the differentiable function R for $0 \le t \le 12$, where $R(t)$ is measured in vehicles per hour and t is the number of hours since 7:00 A.M. ($t = 0$). Values of $R(t)$ for selected values of t are given in the table above.

   (a) Use the data in the table to approximate $R'(5)$. Show the computations that lead to your answer. Using correct units, explain the meaning of $R'(5)$ in the context of the problem.

   (b) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\int_0^{12} R(t)\,dt$. Indicate units of measure.

   (c) On a certain weekend day, the rate at which vehicles cross the bridge is modeled by the function H defined by $H(t) = -t^3 - 3t^2 + 288t + 1300$ for $0 \le t \le 17$, where $H(t)$ is measured in vehicles per hour and t is the number of hours since 7:00 A.M. ($t = 0$). According to this model, what is the average number of vehicles crossing the bridge per hour on the weekend day for $0 \le t \le 12$?

   (d) For $12 < t < 17$, $L(t)$, the local linear approximation to the function H given in part (c) at $t = 12$, is a better model for the rate at which vehicles cross the bridge on the weekend day. Use $L(t)$ to find the time t, for $12 < t < 17$, at which the rate of vehicles crossing the bridge is 2000 vehicles per hour. Show the work that leads to your answer.

Part A (AB or BC): Graphing calculator required
Scoring Guidelines for Question 1 — 9 points

Learning Objectives: CHA-2.D CHA-3.A CHA-3.C CHA-3.F CHA-4.B LIM-5.A

(a) Use the data in the table to approximate $R'(5)$. Show the computations that lead to your answer. Using correct units, explain the meaning of $R'(5)$ in the context of the problem.

Model Solution	Scoring	
$R'(5) \approx \frac{R(6)-R(4)}{6-4} = \frac{3010-3442}{2} = -216$	Approximation using values from table	1 point 2.B
At time $t = 5$ hours (12 P.M.), the rate at which vehicles cross the bridge is decreasing at a rate of approximately 216 vehicles per hour per hour.	Interpretation with units	1 point 3.F 4.B
	Total for part (a)	**2 points**

(b) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\int_0^{12} R(t)\,dt$. Indicate units of measure.

Model Solution	Scoring	
$\int_0^{12} R(t)\,dt \approx 4(R(2)+R(6)+R(10))$	Midpoint sum setup	1 point 1.E
$= 4(3653+3010+1986)$ $= 34{,}596$ vehicles	Approximation using values from the table with units	1 point 2.B 4.B
	Total for part (b)	**2 points**

(c) What is the average number of vehicles crossing the bridge per hour on the weekend day for $0 \le t \le 12$?

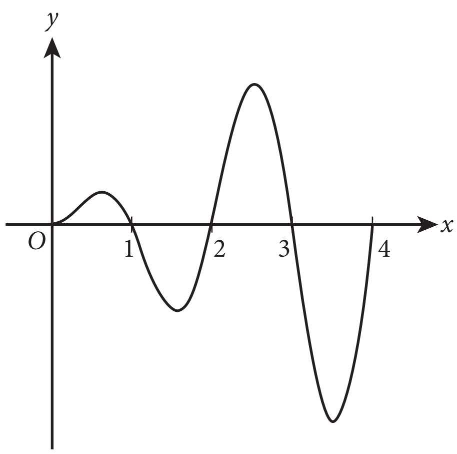
Model Solution	Scoring	
$\frac{1}{12-0}\boxed{\int_0^{12} H(t)\,dt} = 2452$ Definite integral; Answer	Definite integral	1 point 1.D 4.C
	Answer with supporting work	1 point 1.E
	Total for part (c)	**2 points**

(d) Use $L(t)$ to find the time t, for $12 \le t \le 17$, at which the rate of vehicles crossing the bridge is 2000 vehicles per hour. Show the work that leads to your answer.

Model Solution	Scoring	
$L(t) = H(12) - H'(12)(t-12)$ $H(12) = 2596;\ H'(12) = -216$	Slope	1 point 1.E 4.E
$L(t) = 2000$	$L(t) = 2000$	1 point 1.D
$\Rightarrow t = 14.759$	Answer with supporting work	1 point 1.E 4.E
	Total for part (d)	**3 points**

Total for Question 1 — **9 points**

PART B (AB OR BC): Calculator not Permitted



Graph of f'

2. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval [0, 4]. The areas of the regions bounded by the graph of f' and the x-axis on the intervals [0, 1], [1, 2], [2, 3], and [3, 4] are 2, 6, 10, and 14, respectively. The graph of f' has horizontal tangents at $x = 0.6$, $x = 1.6$, $x = 2.5$, and $x = 3.5$. It is known that $f(2) = 5$.

   (a) On what open intervals contained in (0, 4) is the graph of f both decreasing and concave down? Give a reason for your answer.

   (b) Find the absolute minimum value of f on the interval [0, 4]. Justify your answer.

   (c) Evaluate $\int_0^4 f(x)f'(x)dx$.

   (d) The function g is defined by $g(x) = x^3 f(x)$. Find $g'(2)$. Show the work that leads to your answer.

Part A (AB or BC): Calculator not Permitted
Scoring Guidelines for Question 2 — 9 points

Learning Objectives: FUN-3.B FUN-4.A FUN-5.A FUN-6.D

(a) On what open intervals contained in (0,4) is the graph of f both decreasing and concave down? Give a reason for your answer.

Model Solution	Scoring	
The graph of f is decreasing and concave down on the intervals (1, 1.6) and (3, 3.5)	Answer	**1 point** 2.E
because f' is negative and decreasing on these intervals.	Reason	**1 point** 3.E 4.A
	Total for part (a)	**2 points**

(b) Find the absolute minimum value of f on the interval [0, 4]. Justify your answer.

The graph of f' changes from negative to positive only at $x=2$.	Considers $x=2$ as a candidate	**1 point** 3.B
$f(0)=f(2)+\int_2^0 f'(x)\,dx=f(2)-\int_0^2 f'(x)\,dx=5-(2-6)=9$ $f(2)=5$ $f(4)=f(2)+\int_2^4 f'(x)\,dx=5+(10-14)=1$ On the interval [0, 4], the absolute minimum value of f is $f(4)=1$.	Answer with justification	**1 point** 3.E
	Total for part (b)	**2 points**

(c) Evaluate $\int_0^4 f(x)f'(x)\,dx$.

$\int_0^4 f(x)f'(x)\,dx=\frac{1}{2}(f(x))^2\Big\vert_{x=0}^{x=4}$	Antiderivative of the form $a[f(x)]^2$	**1 point** 1.C
$=\frac{1}{2}\left((f(4))^2-(f(0))^2\right)$	Earned the first point and $a=\frac{1}{2}$	**1 point** 1.E
$=\frac{1}{2}(1^2-9^2)=-40$	Answer	**1 point** 2.B
	Total for part (c)	**3 points**

(d) Find $g'(2)$. Show the work that leads to your answer.

$g'(x)=3x^2f(x)+x^3f'(x)$ $g'(2)=3\cdot 2^2f(2)+2^3f'(2)=12\cdot 5+8\cdot 0=60$	Product Rule	**1 point** 1.E
	Answer	**1 point** 2.B
	Total for part (d)	**2 points**
	Total for Question 2	**9 points**

PART A (BC ONLY): Graphing Calculator Required

3. For $0 \le t \le 5$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 1$, the particle is at position $(2, -7)$. It is known that $\frac{dx}{dt} = \sin\left(\frac{t}{t+3}\right)$ and $\frac{dy}{dt} = e^{\cos t}$.

   (a) Write an equation for the line tangent to the curve at the point $(2, -7)$.

   (b) Find the y-coordinate of the position of the particle at time $t = 4$.

   (c) Find the total distance traveled by the particle from time $t = 1$ to time $t = 4$.

   (d) Find the time at which the speed of the particle is 2.5. Find the acceleration vector of the particle at this time.

Part A (BC ONLY): Graphing Calculator Required
Scoring Guidelines for Question 3 — 9 points

Learning Objectives: CHA-3.G FUN-8.B

(a) Write an equation for the line tangent to the curve at the point (2, −7).

Model Solution	Scoring	
$\left.\dfrac{dy}{dx}\right\|_{t=1} = \left.\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}\right\|_{t=1} = \dfrac{e^{\cos 1}}{\sin\left(\frac{1}{4}\right)} = 6.938150$	Slope	**1 point** 1.C 4.E
An equation for the line tangent to the curve at the point (2, −7) is $y = -7 + 6.938(x-2)$.	Tangent line equation	**1 point** 1.D
	Total for part (a)	**2 points**

(b) Find the y-coordinate of the position of the particle at time $t = 4$.

Model Solution	Scoring	
$y(4) = -7 + \int_1^4 \dfrac{dy}{dt}\,dt = -5.006667$	Definite integral	**1 point** 1.D 4.C
The y-coordinate of the position of the particle at time $t = 4$ is −5.007 (or −5.006).	Answer	**1 point** 2.B
	Total for part (b)	**2 points**

(c) Find the total distance traveled by the particle from time $t = 1$ to time $t = 4$.

Model Solution	Scoring	
$\int_1^4 \sqrt{\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2}\,dt = 2.469242$	Definite integral	**1 point** 1.D 4.C
The total distance traveled by the particle from time $t = 1$ to time $t = 4$ is 2.469.	Answer	**1 point** 1.E 4.E
	Total for part (c)	**2 points**

(d) Find the time at which the speed of the particle is 2.5. Find the acceleration vector of the particle at this time.

Model Solution	Scoring	
$\sqrt{\left(\dfrac{dx}{dt}\right)^2 + \left(\dfrac{dy}{dt}\right)^2} = 2.5 \Rightarrow t = 0.415007$	Speed equation	**1 point** 1.D 4.C
The speed of the particle is 2.5 at time $t = 0.415$.	Value of t	**1 point** 1.E 4.E
The acceleration vector of the particle at time $t = 0.415$ is: $\langle x''(0.415), y''(0.415)\rangle = \langle 0.255, -1.007\rangle$ (or $\langle 0.255, -1.006\rangle$).	Acceleration vector	**1 point** 1.E 4.E
	Total for part (d)	**3 points**

	Total for Question 3	**9 points**

PART B (BC ONLY): Calculator not Permitted

4. The Maclaurin series for the function *f* is given by
$f(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^k}{k^2} = x - \frac{x^2}{4} + \frac{x^3}{9} - \cdots$ on its interval of convergence.

(a) Use the ratio test to determine the interval of convergence of the Maclaurin series for *f*. Show the work that leads to your answer.

(b) The Maclaurin series for *f* evaluated at $x = \frac{1}{4}$ is an alternating series whose terms decrease in absolute value to 0. The approximation for $f\left(\frac{1}{4}\right)$ using the first two nonzero terms of this series is $\frac{15}{64}$. Show that this approximation differs from $f\left(\frac{1}{4}\right)$ by less than $\frac{1}{500}$.

(c) Let *h* be the function defined by $h(x) = \int_0^x f(t)\,dt$. Write the first three nonzero terms and the general term of the Maclaurin series for *h*.

Part B: (BC ONLY): Calculator not Permitted
Scoring Guidelines for Question 4 — 9 points

Learning Objectives: LIM-7.A LIM-7.B LIM-8.D LIM-8.G

(a) Use the ratio test to determine the interval of convergence of the Maclaurin series for *f*. Show the work that leads to your answer.

Model Solution	Scoring	
$\lim_{k\to\infty}\left|\dfrac{\dfrac{(-1)^{k+2}x^{k+1}}{(k+1)^2}}{\dfrac{(-1)^{k+1}x^k}{k^2}}\right| = \lim_{k\to\infty}\dfrac{k^2}{(k+1)^2}\lvert x\rvert = \lvert x\rvert$	Sets up ratio	**1 point** 3.B
	Computes limit of ratio	**1 point** 1.E 4.C
$\lvert x\rvert < 1$ The series converges for $-1 < x < 1$.	Identifies interior or interval of convergence	**1 point** 3.D
When $x = -1$, the series is $\sum_{k=1}^{\infty}\frac{-1}{k^2}$. This is a convergent *p*-series.	Considers both endpoints	**1 point** 1.D
When $x = 1$, the series is $\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2}$. This series converges by the alternating series test. The interval of convergence of the Maclaurin series for *f* is $-1 \le x \le 1$.	Analysis and interval of convergence	**1 point** 3.D

Total for part (a) — **5 points**

(b) Show that this approximation differs from $f\left(\frac{1}{4}\right)$ by less than $\frac{1}{500}$.

Model Solution	Scoring	
$\left|f\left(\frac{1}{4}\right)-\frac{15}{64}\right| < \dfrac{\left(\frac{1}{4}\right)^3}{9} = \frac{1}{576}$	Uses third term as error bound	**1 point** 3.D
$\frac{1}{576} < \frac{1}{500}$	Error bound	**1 point** 3.E

Total for part (b) — **2 points**

(c) Write the first three nonzero terms and the general term of the Maclaurin series for *h*.

Model Solution	Scoring	
$h(x) = \int_0^x f(t)\,dt = \frac{x^2}{2} - \frac{x^3}{12} + \frac{x^4}{36} - \cdots + \frac{(-1)^{k+1}x^{k+1}}{(k+1)k^2} + \cdots$ First three nonzero terms; General term	First three nonzero terms	**1 point** 1.D
	General term	**1 point** 1.D 4.C

Total for part (c) — **2 points**

Total for Question 4 — **9 points**