PROFESSIONAL DEVELOPMENT

AP® Calculus
Volumes of Solids of Revolution

Curriculum Module

CollegeBoard

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The College Board

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Contents

Volumes of Solids of Revolution: Rotating About a Line Other than the *x*- or *y*-Axis

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Questions involving the area of a region between curves, and the volume of the solid formed when this region is rotated about a horizontal or vertical line, appear regularly on both the AP® Calculus AB and BC exams. Student performance on this problem is generally quite strong except when the solid is formed using a line of rotation other than the *x*- or y-axis. Students have difficulty finding volumes of solids with a line of rotation other than the *x*- or y-axis. My visual approach to these problems develops an understanding of how each line of rotation affects the radius of the solid. The approach I use (equivalent to the washer method) in the following four examples is what I have found to be most effective in my own classroom.

The four examples I have chosen include a line of rotation below the region to be rotated, above the region to be rotated, to the left of the region to be rotated, and to the right of the region to be rotated.

The disk method is used in all four examples. All rotations result in a solid with a hole. The “washer” method uses one integral to find the volume of the solid. I use two integrals, finding the answer as the volume of a solid minus the volume of the hole. I have found that when they set up these problems using two integrals, my students understand better what each part of the integral, especially the integrand, represents. Thus, they are successful with this type of problem.

Example 1. Line of Rotation Below the Region to Be Rotated

Let's start by looking at the solid (with a hole) we get when we rotate the region bounded by $y = \sqrt{x+2}$ and $y = e^x$ about the line $y = -2$.

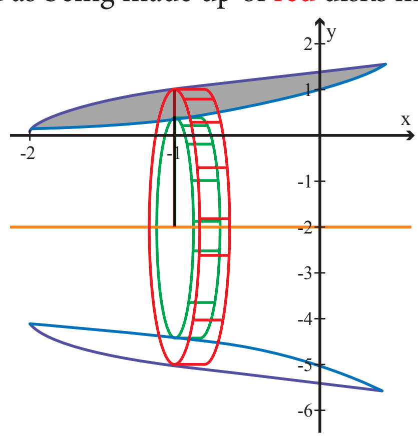
The two curves intersect at the points $(-1.980974, 0.13793483)$ and $(0.44754216, 1.5644623)$.

Color will be used to help visualize the problem situation. Using the same color for the same feature consistently from problem to problem helps students follow the process as they are learning to solve these problems. Orange will be used for the line of rotation. Purple is used for the top curve and blue for the bottom curve.

Drawing the disks used to solve the problem helps students follow the steps in the setup and ties the calculus "language" to what we are doing. I verbalize for my students what I have written here for you while I draw the diagrams on the board. You may want to try drawing the disks as I describe the procedure for you. All graphs were created using the free program Winplot. My favorite Winplot tutorial is at http://spot.pcc.edu/~ssimonds/winplot/.

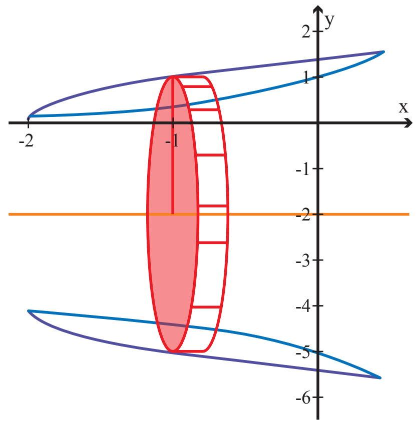
I find that reflecting the original shaded region across the line of rotation helps me make a drawing of the solid.

We will think of our solid as being made up of red disks minus green disks. The following



graph is a representation.

First draw a typical red disk. Start by drawing a radius from the orange line of rotation through the original shaded region until you get to the outside of the region. Stop at this purple curve. *Note:* You can draw the disk in other ways. However, if the radius is not drawn in this way, students often have difficulty getting the expression for the radius.



Starting from the orange line of rotation, we move up (vertically) through the shaded region. Vertical is the y direction, so the red radius involves "y". Let y_{purple} be the y-coordinate of a point on the purple curve, and picture y_{purple} as running vertically from the x-axis to the purple curve. From the orange line of rotation, we have to move two units up to get to the x-axis. Then we move from the x-axis to the purple curve to complete the radius. Therefore, the radius of each red disk is $r = 2 + y_{\text{purple}}$.

The volume of a disk is just the volume of a cylinder where the thickness of the disk is the height of the cylinder. So the volume of one disk is $\pi r^2 \cdot thickness$. In this case, since the radius of the red disk is vertical, the thickness (height) is horizontal. And since horizontal is the x direction, the thickness is Δx. Since $y_{\text{purple}} = \sqrt{x+2}$, we have:

$$\text{Volume of each red disk} = \pi r^2 \Delta x = \pi\,(2 + y_{\text{purple}})^2\,\Delta x = \pi\left(2+\sqrt{x+2}\right)^2 \Delta x\,.$$

We now stack up n red disks from the point of intersection $x = -1.980974$ on the left to the other point of intersection $x = 0.44754216$ on the right.

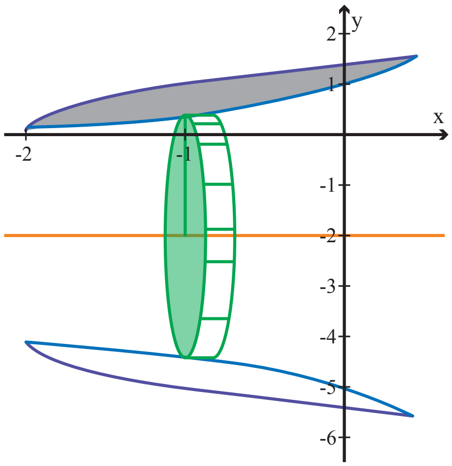
The sum of the volumes of these n red disks is given by $\sum_{k=1}^{n} \pi\left(2+\sqrt{x_k+2}\right)^2 \Delta x$.

The larger the number of disks and the thinner each disk, the smoother the stack of red disks will be. Thus, to obtain a perfectly smooth red solid, we let $n \to \infty$ and $\Delta x \to 0$. The volume of this red solid is the limit of the sum of the volumes of n red disks as $n \to \infty$ (and $\Delta x \to 0$).

Volume of red solid =

$$\lim_{n\to\infty}\left(\sum_{k=1}^{n} \pi\left(2+\sqrt{x_k+2}\right)^2 \Delta x\right) = \int_{-1.980974}^{0.44754216} \pi\left(2+\sqrt{x+2}\right)^2 dx = 71.983363.$$

The points of intersection can be reported to three decimal places by either rounding or truncating, but we **cannot** use these rounded answers for calculation because our final answer may not be accurate to three decimal places as needed. The easiest thing to do is to store these values in the calculator and then retrieve them when evaluating the integral.



We now will find the volume of the green disks creating the hole. To draw a typical green disk, first draw the radius from the orange line of rotation until you hit the edge of the original shaded region. Stop at the blue curve. Starting at the orange line of rotation, we move up (vertically) to the edge of the shaded region. Vertical is the y direction, so the green radius involves "y". Note that y_{blue} goes only from the x-axis to the blue curve. From the orange line of rotation, we have to move two units up to get to the x-axis. Then we move from the x-axis to the curve to complete the radius. Therefore, the radius of each green disk is $r = 2 + y_{\text{blue}}$.

The volume of one disk is $\pi r^2 \cdot thickness$. Once again, since the radius of the green disk is vertical, the thickness (height) is horizontal. And since horizontal is the x direction, the thickness is Δx. Since $y_{\text{blue}} = e^x$, we have:

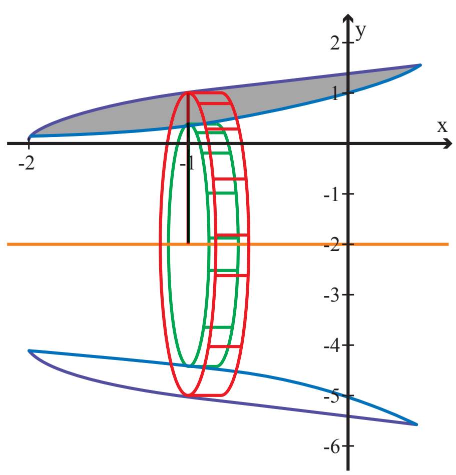
$$\text{Volume of each green disk} = \pi\, r^2 \Delta x \;=\; \pi\,(2 + y_{\text{blue}})^2\, \Delta x \;=\; \pi\left(2 + e^x\right)^2 \Delta x\,.$$

We now stack up n green disks from the point of intersection $x = -1.980974$ on the left to the other point of intersection $x = 0.44754216$ on the right.

The sum of the volumes of these n green disks is given by $\displaystyle\sum_{k=1}^{n} \pi\left(2 + e^{x_k}\right)^2 \Delta x$.

To obtain a perfectly smooth green solid, we let $n \to \infty$ and $\Delta x \to 0$. The volume of this green solid is the limit of the sum of the volumes of n green disks as $n \to \infty$ (and $\Delta x \to 0$).

$$\text{Volume of green solid} = \lim_{n\to\infty}\left(\sum_{k=1}^{n} \pi\left(2 + e^{x_k}\right)^2 \Delta x\right) = \int_{-1.980974}^{0.44754216} \pi\left(2 + e^x\right)^2 dx = 52.258610.$$



So, the volume of the solid we get when we rotate the region bounded by $y=\sqrt{x+2}$ and $y=e^x$ about the line $y=-2$ is:

VOLUME of red solid – VOLUME of green solid

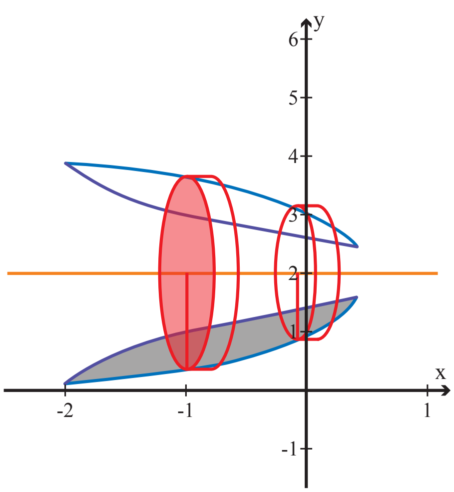
$$= \int_{-1.980974}^{0.44754216} \pi\left(2+\sqrt{x+2}\right)^2 dx - \int_{-1.980974}^{0.44754216} \pi\left(2+e^x\right)^2 dx = 19.724 \text{ or } 19.725.$$

I have found that students are better able to focus on the idea of the solid of revolution as a difference of two solids if I do not emphasize the role of limit notation in this process. Therefore, in each example that follows, I will show how to write the volume of each red disk and each green disk, and then will refer to the definite integral as the sum of infinitely many disks.

Example 2. Line of Rotation Above the Region to Be Rotated

Now let's rotate the same region about $y=2$. We're rotating the region bounded by $y=\sqrt{x+2}$ and $y=e^x$ about the line $y=2$.

The solid will be made up of red disks minus green disks.



To draw the red disks that make up the solid, first draw the radius from the orange line of rotation through the original shaded region until you get to the outside of the region. Stop at the blue curve. From the orange line of rotation through the shaded region, we move down (vertically). Vertical is the y direction, so the red radius involves "y". Now y_{blue} goes from the x-axis to the blue curve. This is not the red radius.

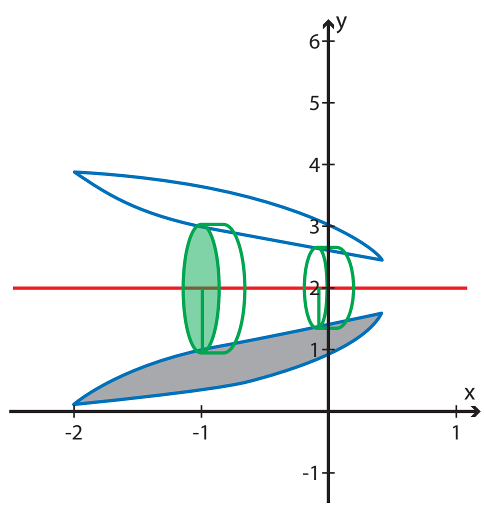
Notice that y_{blue} + radius = 2. Therefore, the radius r of each red disk is $r = 2 - y_{\text{blue}}$. The radius of the red disk is vertical; the thickness (height) is horizontal. Since horizontal is the x direction, the thickness is Δx, and the volume is $\pi\, r^2 \cdot thickness$.
Since $y_{\text{blue}} = e^x$, we have:

$$\text{Volume of each red disk} = \pi\, r^2 \Delta x \;=\; \pi\,(2 - y_{\text{blue}})^2\, \Delta x \;=\; \pi\left(2 - e^x\right)^2 \Delta x\,.$$

As in Example 1, the red disks are stacked up from $x = -1.980974$ on the left to $x = 0.44754216$ on the right. We use a definite integral to add up an infinite number of these red disks. This gives us:

$$\text{Volume of red solid} = \int_{-1.980974}^{0.44754216} \pi\left(2 - e^x\right)^2 dx \;=\; 16.406065.$$

We will now find the volume of the green disks creating the hole.



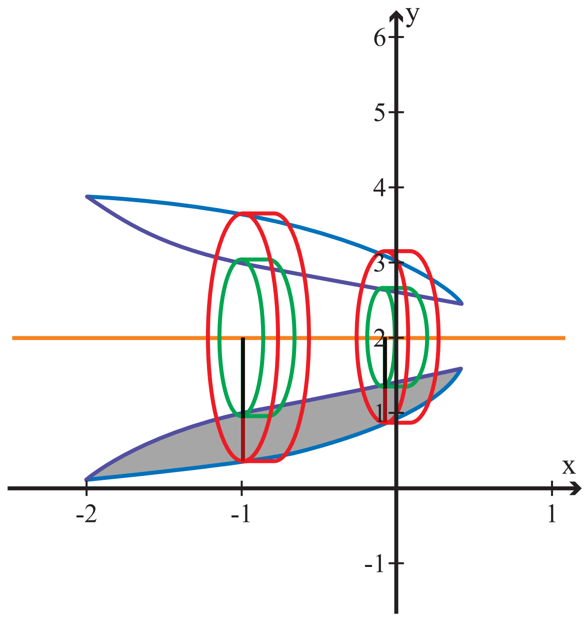
To draw the green disks that create the hole, first draw the radius from the orange line of rotation until you hit the edge of the original shaded region. Stop at the purple curve. From the orange line of rotation, we move down (vertically) to the edge of the shaded region. Vertical is the y direction, so the green radius involves "y". Note that y_{purple} goes from the x-axis to the purple curve. This is not the green radius. Notice that y_{purple} + radius = 2. Therefore, the radius r of each green disk is $r = 2 - y_{\text{purple}}$.

The volume of one disk is $\pi\, r^2 \cdot \textit{thickness}$. Once again, since the radius of the green disk is vertical, the thickness (height) is horizontal. And since horizontal is the x direction, the thickness is Δx. Since $y_{\text{purple}} = \sqrt{x+2}$, we have:

$$\text{Volume of each green disk} = \pi r^2 \Delta x \;=\; \pi\,(2 - y_{\text{purple}})^2 \Delta x \;=\; \pi\left(2 - \sqrt{x+2}\right)^2 \Delta x\,.$$

Again, the green disks are stacked up from $x = -1.980974$ on the left to $x = 0.44754216$ on the right, and we use a definite integral to add up an infinite number of these green disks. This gives us:

$$\text{Volume of green solid} = \int_{-1.980974}^{0.44754216} \pi\left(2 - \sqrt{x+2}\right)^2 dx \;=\; 7.870360.$$



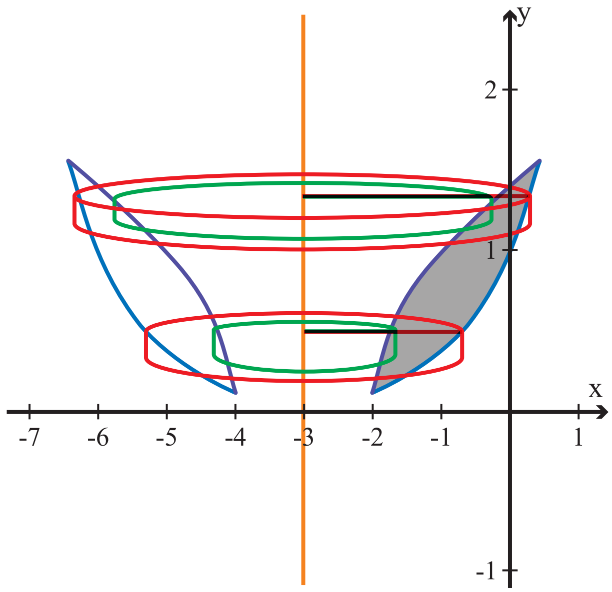
So, the volume of the solid we get when we rotate the region bounded by $y = \sqrt{x+2}$ and $y = e^x$ about the line $y = 2$ is:

VOLUME of red solid – VOLUME of green solid

$$= \int_{-1.980974}^{0.44754216} \pi\left(2 - e^x\right)^2 dx - \int_{-1.980974}^{0.44754216} \pi\left(2 - \sqrt{x+2}\right)^2 dx = 8.5357 = 8.535 \text{ or } 8.536.$$

Example 3. Line of Rotation to the Left of the Region to Be Rotated

Now let's rotate around the vertical line $x = -3$. We will rotate the region bounded by $y = \sqrt{x+2}$ and $y = e^x$ about the line $x = -3$.



Our solid will be made up of red disks minus green disks. To draw the red disks that make up the solid, first draw the radius from the orange line of rotation through the original shaded region until you get to the outside of the region. Stop at the blue curve. From the orange line of rotation, we move right (horizontally) through the shaded region. Horizontal is the x direction, so the red radius involves "x". Now x_{blue} is the x-coordinate of a point on the blue curve, and can be pictured as running horizontally from the y-axis to the blue curve. This is not the radius.

Notice that, in the first quadrant, radius = $3 + x_{\text{blue}}$ where x_{blue} is positive. In the second quadrant,

radius = 3 – the distance from the y-axis to the blue curve.

But x_{blue} is negative, so this distance is $|x_{\text{blue}}| = -x_{\text{blue}}$ and we have radius $= 3 - (-x_{\text{blue}}) = 3 + x_{\text{blue}}$. Therefore, the radius r of each red disk is $r = 3 + x_{\text{blue}}$.

The volume of one disk is $\pi r^2 \cdot thickness$. Since the radius of the red disk is horizontal, the thickness (height) is vertical. And since vertical is the y direction, the thickness is Δy. Since $y_{\text{blue}} = e^x$, then $x_{\text{blue}} = \ln y$ and we have:

$$\text{Volume of each red disk} = \pi r^2 \Delta y = \pi (3 + x_{\text{blue}})^2 \Delta y = \pi \left(3 + \ln y\right)^2 \Delta y.$$

Once again, we use a definite integral to add up an infinite number of these red disks. The red disks are stacked up from $y = 0.1379348256$ on the bottom to $y = 1.564462259$ on the top. This gives us:

$$\text{Volume of red solid} = \int_{0.1379348256}^{1.564462259} \pi \left(3 + \ln y\right)^2 dy.$$

We will now find the volume of the green disks creating the hole. To draw the green disks that create the hole, first draw the radius from the orange line of rotation until you hit the edge of the original shaded region. Stop at the purple curve. From the orange line of rotation, we move right (horizontally) through the shaded region. Horizontal is the x direction, so the green radius involves x. Now, x_{purple} is the x-coordinate of a point on the purple curve and can be pictured as running horizontally from the y-axis to the purple curve. This is not the radius. Notice again that in the first quadrant, radius = 3 + x_{purple} where x_{purple} is positive. In the second quadrant,

radius = 3 – the distance from the y-axis to the purple curve.

But x_{purple} is negative, so this distance is $|x_{\text{purple}}| = -x_{\text{purple}}$ and we have radius $= 3 - (-x_{\text{purple}}) = 3 + x_{\text{purple}}$. Therefore, the radius r of each green disk is $r = 3 + x_{\text{purple}}$. Since $y_{\text{purple}} = \sqrt{x+2}$, then $x_{\text{purple}} = y^2 - 2$ and we have:

$$\text{Volume of each green disk} = \pi r^2 \Delta y = \pi\,(3 + x_{\text{purple}})^2\, \Delta y = \pi\left(3+\left(y^2-2\right)\right)^2 \Delta y\,.$$

The volume of one disk is $\pi\, r^2 \cdot \textit{thickness}$. Since the radius of the green disk is horizontal, the thickness (height) is vertical. And since vertical is the y direction, the thickness is Δy.

Again, we use a definite integral to add up an infinite number of these green disks stacked up from $y = 0.1379348256$ on the bottom to $y = 1.564462259$ on the top. This gives us:

$$\text{Volume of green solid} = \int_{0.1379348256}^{1.564462259} \pi\left(3+\left(y^2-2\right)\right)^2 dy\,.$$

So, the volume of the solid we get when we rotate the region bounded by $y=\sqrt{x+2}$ and $y=e^x$ about the line $x=-3$ is:

VOLUME of red solid – VOLUME of green solid

$$= \int_{0.1379348256}^{1.564462259} \pi\left(3+\ln y\right)^2 dy \;-\; \int_{0.1379348256}^{1.564462259} \pi\left(3+\left(y^2-2\right)\right)^2 dy = 15.5397 = 15.539 \text{ or } 15.540.$$

Example 4. Line of Rotation to the Right of the Region to Be Rotated

Let's now rotate about the line $x = 1$. We will rotate the region bounded by $y = \sqrt{x+2}$ and $y = e^x$ about the line $x = 1$.

Our solid will be made up of red disks minus green disks. To draw the red disks that make up the solid, first draw the radius from the orange line of rotation through the original shaded region until you get to the outside of the region. Stop at the purple curve. From the orange line of rotation, we move left (horizontally) through the shaded region. Horizontal is the x direction, so the red radius involves "x". Note that x_{purple} goes from the y-axis to the (purple) curve. This is not the radius. Notice that in the first quadrant, radius + $x_{\text{purple}} = 1$ where x is positive. Or radius $= 1 - x_{\text{purple}}$. In the second quadrant,

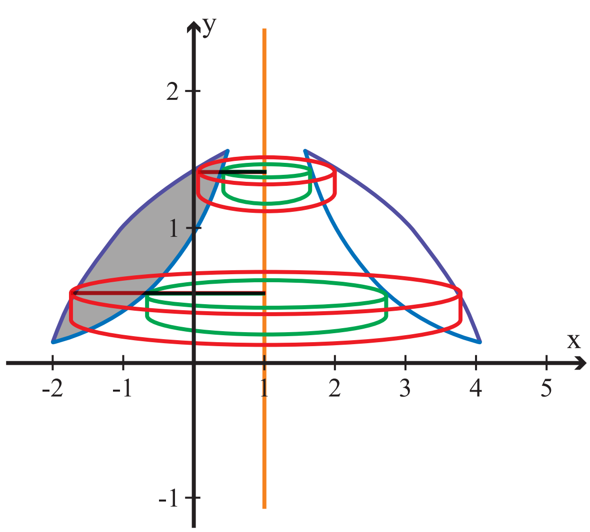
radius = 1 + the distance from the y-axis to the purple curve.

But x_{purple} is negative, so this distance is $|x_{\text{purple}}| = -x_{\text{purple}}$ and we have radius $= 1 + (-x_{\text{purple}}) = 1 - x_{\text{purple}}$. This is the same expression for the radius as in the first quadrant. Therefore, radius $= 1 - x_{\text{purple}}$.

The volume of one disk is $\pi r^2 \cdot thickness$. Since the radius of the red disk is horizontal, the thickness (height) is vertical. And since vertical is the y direction, the thickness is Δy. We use a definite integral to add up an infinite number of these red disks stacked up from $y = 0.1379348256$ on the bottom to $y = 1.564462259$ on the top. This gives us:

$$\text{Volume of red solid} = \int_{0.1379348256}^{1.564462259} \pi\left(1-\left(y^2-2\right)\right)^2 dy\,.$$

We will now find the volume of the green disks creating the hole.



To draw the green disks that create the hole, first draw the radius from the orange line of rotation until you hit the edge of the original shaded region. Stop at the blue curve. From the orange line of rotation, we move left (horizontally) through the shaded region. Horizontal is the x direction, so the green radius involves "x". Now x_{blue} goes from the y-axis to the blue curve. This is not the radius. Notice that in the first quadrant, radius + $x_{\text{blue}} = 1$ where x_{blue} is positive, or radius $= 1 - x_{\text{blue}}$.

In the second quadrant,

radius = 1 + the distance from the y-axis to the blue curve.

But x_{blue} is negative, so this distance is $|x_{\text{blue}}| = -x_{\text{blue}}$ and we have

radius = $1 + (-x_{\text{blue}}) = 1 - x_{\text{blue}}$. This is the same expression for the radius as in the first quadrant. Therefore, radius = $1 - x_{\text{blue}}$.

The volume of one disk is $\pi\, r^2 \cdot \textit{thickness}$. Since the radius of the green disk is horizontal, the thickness (height) is vertical. And since vertical is the y direction, the thickness is Δy.

We use a definite integral to add up an infinite number of these green disks stacked up from $y = 0.1379348256$ on the bottom to $y = 1.564462259$ on the top. Since $y_{\text{blue}} = e^x$, then $x_{\text{blue}} = \ln y$ and we have:

$$\text{Volume of green solid} = \int_{0.1379348256}^{1.564462259} \pi\left(1 - \ln y\right)^2 dy.$$

So, the volume of the solid we get when we rotate the region bounded by $\underline{y = \sqrt{x+2}}$ and $\underline{y = e^x}$ about the line $\underline{x = 1}$ is:

VOLUME of red solid – VOLUME of green solid

$$= \underline{\int_{0.1379348256}^{1.564462259} \pi\left(1 - \left(y^2 - 2\right)\right)^2 dy} - \underline{\int_{0.1379348256}^{1.564462259} \pi\left(1 - \ln y\right)^2 dy} = 12.72067 = 12.720 \text{ or } 12.721.$$

If the x- or y-axis is the line of rotation, the problem can be done in the same way as the four examples shown here. The radius will just be the appropriate x or y value. Yes, there are patterns in the integrand depending on the line of rotation. But there is no reason to memorize patterns and take the chance of making a mistake. A drawing is all that is needed to get a problem of this type set up correctly.

AP Examination Questions

Problems from any calculus book can be used to practice this concept. Be careful, however, that you don't assign problems where the line of rotation goes through the interior of the shaded region. This situation can be interesting but also can involve some tedious algebra. You will notice when looking at the problems in the list below that there has never been an AP question where the line of rotation goes through the interior of the shaded region.

Since 1997 the following free-response questions from the AP Calculus examinations have involved revolving a region about a horizontal or vertical line.

1997	AB2d	line of rotation x-axis
1998	AB/BC1c	line of rotation x-axis
	AB/BC1d	line of rotation x-axis
1999	AB/BC2b	line of rotation x-axis
	AB/BC2c	**line of rotation $y = k$**
2000	AB/BC1b	line of rotation x-axis
2001	AB1c	line of rotation x-axis
2002	**AB/BC1b**	**line of rotation $y = 4$**
2002B	AB1b	line of rotation x-axis
	BC3b	line of rotation x-axis
2003	**AB/BC1b**	**line of rotation $y = 1$**
2003B	AB1c	line of rotation x-axis
2004	**AB/BC2**	**line of rotation $y = 2$**

2004B	**AB1c**	**line of rotation $y = 3$**
	AB1d	**line of rotation $x = 4$**
	BC5b	line of rotation x-axis
2005	**AB/BC1c**	**line of rotation $y = -1$**
2005B	AB1b	line of rotation x-axis
	BC6b	line of rotation x-axis
2006	**AB/BC1b**	**line of rotation $y = -3$**
	AB/BC1c	line of rotation y-axis
2006B	**AB/BC1b**	**line of rotation $y = -2$**
2007	AB/BC1b	line of rotation x-axis
2007B	**AB/BC1c**	**line of rotation $y = 1$**
2008B	**AB1b**	**line of rotation $x = -1$**
2008B	BC4b	line of rotation x-axis

For a complete listing of free-response questions, you can access apcentral.collegeboard.com/apc/public/exam/exam_questions/index.html.

Practice Problems

1. Find the volume of the solid resulting from rotating the region enclosed by $y = x^2$ and $y = x^3$ about the vertical line $x = 1$.
2. Find the volume of the solid resulting from rotating the region enclosed by $y = \sin x$ and $y = x^2$ about the horizontal line $y = 1$.
3. Find the volume of the solid resulting from rotating the region enclosed by $y = x^2$ and $y = x^3$ about the vertical line $x = -1$.
4. Find the volume of the solid resulting from rotating the region enclosed by $y = \sin x$ and $y = x^2$ about the horizontal line $y = -1$.

Solutions to Practice Problems

1. The radius of each large (red) disk is horizontal.

   Red radius = $1 - x_{\text{purple (outside)}} = 1 - \sqrt{y}$.

   Volume of the red solid = $\int_0^1 \pi\left(1-\sqrt{y}\right)^2 dy$.

   The radius of each small (green) disk is also horizontal.

   Green radius = $1 - x_{\text{blue (inside)}} = 1 - \sqrt[3]{y}$.

   Volume of the green solid = $\int_0^1 \pi\left(1-\sqrt[3]{y}\right)^2 dy$.

   Volume of the solid of revolution = $\int_0^1 \pi\left(1-\sqrt{y}\right)^2 dy - \int_0^1 \pi\left(1-\sqrt[3]{y}\right)^2 dy = 0.209$.

2. The radius of each large (red) disk is vertical.

   Red radius = $1 - y_{\text{blue (outside)}} = 1 - x^2$.

   Volume of the red solid = $\int_0^{0.8767} \pi\left(1-x^2\right)^2 dx$.

   The radius of each small (green) disk is also vertical.

   Green radius = $1 - y_{\text{purple (inside)}} = 1 - \sin x$.

   Volume of the green solid = $\int_0^{0.8767} \pi\left(1-\sin x\right)^2 dx$.

   Volume of the solid of revolution = $\int_0^{0.8767} \pi\left(1-x^2\right)^2 dx - \int_0^{0.8767} \pi\left(1-\sin x\right)^2 dx = 0.573$.

3. The radius of each large (red) disk is horizontal.

   Red radius = $1 + x_{\text{blue (outside)}} = 1 + \sqrt[3]{y}$.

   Volume of the red solid = $\int_0^1 \pi\left(1+\sqrt[3]{y}\right)^2 dy$.

   The radius of each small (green) disk is also horizontal.

   Green radius = $1 + x_{\text{purple (inside)}} = 1 + \sqrt{y}$.

   Volume of the green solid = $\int_0^1 \pi\left(1+\sqrt{y}\right)^2 dy$.

   Volume of the solid of revolution = $\int_0^1 \pi\left(1+\sqrt[3]{y}\right)^2 dy - \int_0^1 \pi\left(1+\sqrt{y}\right)^2 dy = 0.838$.

4. The radius of each large (red) disk is vertical.

   Red radius = $1 + y_{\text{purple (top)}} = 1 + \sin x$.

   Volume of the red solid = $\int_0^{0.8767} \pi\left(1+\sin x\right)^2 dx$.

   The radius of each small (green) disk is also vertical.

   Green radius = $1 + y_{\text{blue (bottom)}} = 1 + x^2$.

   Volume of the green solid = $\int_0^{0.8767} \pi\left(1+x^2\right)^2 dx$.

   Volume of the solid of revolution = $\int_0^{0.8767} \pi\left(1+\sin x\right)^2 dx - \int_0^{0.8767} \pi\left(1+x^2\right)^2 dx = 1.131$ or 1.132.

About the Contributor

Catherine Want was a high school math teacher for 33 years and has taught every level of mathematics. She was with Grossmont Union High School District in San Diego County, California, for 15 years and Irvine Unified School District in Irvine, California, for 18 years. Catherine also spent a year as a Fulbright Exchange Teacher in England.

Catherine taught both AP Calculus AB and AP Calculus BC for 25 years and AP Statistics for nine years. She was an AP Calculus Exam Reader for seven years and a Table Leader for an additional six years. As a College Board consultant, she has presented at workshops in California, Oregon, Washington, Utah, Arizona, Texas, Kentucky, Florida, New York, Alaska and Hawaii, and also has conducted workshops overseas in Saipan, Thailand, Indonesia, Germany and India. Catherine received commendation from the University of California, Irvine; was a Siemens Teacher Excellence Award winner in 2003; and was named a Teacher of Merit by INTEL in 2005

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