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AP® Physics

Multiple Representations of Knowledge: Mechanics and Energy

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Introduction

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The objective of these Special Focus Materials is to present a detailed overview of the use of multiple representations in various topics of mechanics and energy (thermodynamics). The use of multiple representations of data and phenomena in physics is a powerful strategy to help students develop a deeper understanding of concepts and effective problem-solving skills. Some of the most commonly used multiple representations in physics are verbal descriptions, mathematical interpretations, pictures, graphs, motion diagrams, free-body diagrams, circuit diagrams, and geometric optics ray tracing.

The first article, by Eugenia Etkina, David Rosengrant, and Alan Van Heuvelen, describes a learning strategy centered on multiple representations for kinematics and dynamics. The authors first give a general outline of the use of multiple representations before focusing on a detailed application of various representations in linear kinematics and both linear and circular motion dynamics. Special emphasis is given to qualitative analysis with the use of motion diagrams and free-body diagrams. The authors suggest various pedagogical approaches that include examples of formative assessments. In the final part of this article, the authors include physics education research data on the implementation of multiple representations in summative assessments at the college level.

The second article, by Randall Knight, focuses on the topic of energy as it applies to mechanics and the first law of thermodynamics. The article is divided into four lessons that provide guidance to teachers on how to introduce and develop the concepts of energy and work with the aid of verbal descriptions, energy bar charts, ranking tasks, and *pV* diagrams. The first lesson deals with conservation of energy in mechanical systems. Lesson two presents the connection between work and thermal energy. The last two lessons include a detailed study of the first law of thermodynamics and energy flow in heat engines. Examples of formative assessments are given in each section.

Using Multiple Representations to Improve Student Learning in Mechanics

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Introduction

The conceptual knowledge in physics courses is often found in an abstract symbolic form. The symbols have precise meanings and must be combined in rules that are used correctly. In contrast, the human mind relates best to picture-like representations that emphasize qualitative features but not detailed, precise information.[1] If we want students to learn the symbolic representations used in the practice of physics (for example, the mathematical descriptions of processes), we have to link these abstract ways of describing the world to more concrete descriptions.

This article describes a learning strategy that emphasizes multiple ways of representing processes for the concepts of kinematics and dynamics. We start with an overview of this multiple representation strategy. We then look in greater detail at how the strategy can be integrated into instruction in kinematics, linear dynamics, and circular motion dynamics. The discussion will provide many examples of formative assessments to help teachers evaluate and modify their instruction if necessary, and for students to evaluate and modify their learning if necessary. Finally, assessment

1. J. H. Larkin and H. Simon, "Why a Diagram Is (Sometimes) Worth Ten Thousand Words," *Cognitive Science* 11 (1997): 65–99.

outcomes where the strategy has been used are presented, along with suggestions for summative assessments that teachers can use to evaluate the learning of their students.

Overview of Multiple Representations for Kinematics and Dynamics

There is considerable research to show that students from high school to honors college manage to solve problems with little understanding of the concepts being used.[2] One difficulty is that the symbols in the mathematical equations have little meaning for the students.[3] One way to address this difficulty is to have students learn to represent physical processes in multiple ways and learn to convert from one representation to another in any direction.[4] This helps students make connections between concrete ways of representing a process (pictures and diagrams) and more abstract ways of representing the same processes (graphs and equations). Additional literature on translating between representations and student learning can be found in Appendix A.

2. E. Mazur, *Peer Instruction: A User's Manual* (Upper Saddle River, New Jersey: Prentice Hall, 1997).
3. A. Van Heuvelen, "Learning to Think Like a Physicist: A Review of Research-Based Instructional Strategies," *American Journal of Physics* 59 (1991a): 891–97; A. Van Heuvelen, "Overview, Case Study Physics," *American Journal of Physics* 59 (1991b): 898–907.
4. Xueli Zou, *The Use of Multiple Representations and Visualizations in Student Learning of Introductory Physics*. Unpublished Ph.D. dissertation, The Ohio State University, 2000.

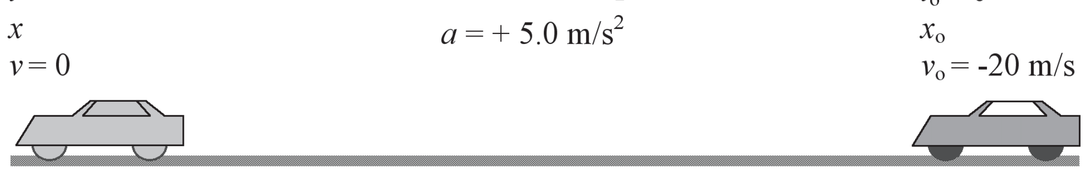
Kinematics Representations

FIGURE 1. A KINEMATICS PROCESS IS REPRESENTED IN MULTIPLE WAYS

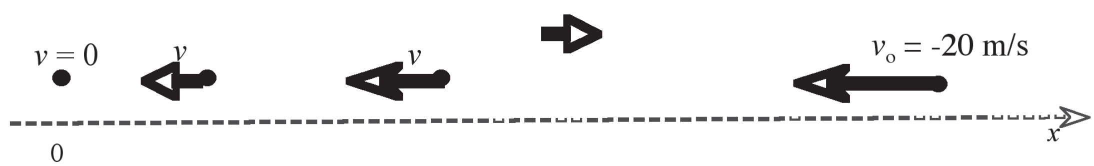
Word Description

A car initially moving west at speed 20 m/s slows to a stop with an acceleration of magnitude 5.0 m/s^2.

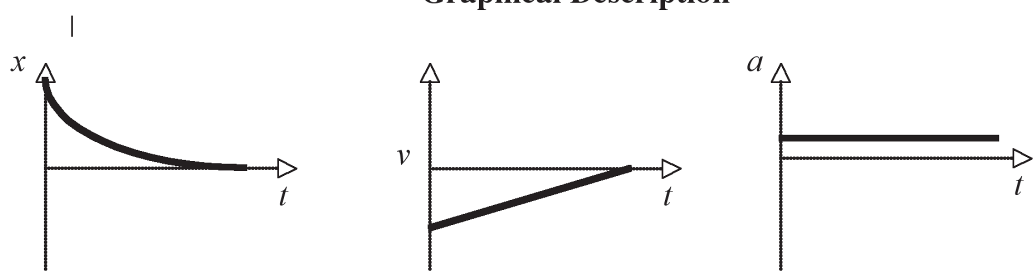
Picture Description



Diagrammatic Description



Graphical Description



Mathematical Description

$$0 - (-20\ \text{m/s}) \ = \ (+\ 5.0\ \text{m/s}^2)\ t$$

$$x - x_o = (-20\ \text{m/s})\ t + 0.5\ (+\ 5.0\ \text{m/s}^2)\ t^2$$

Figure 1 shows a multiple-representation description of a moderately simple one-dimensional kinematics problem: describing the motion of a car as it slows to a stop. We use each successive representation to help construct the next. We first convert the words in the problem statement to a sketch where we include known information and identify the unknowns.

This is often the most difficult task for students. The mind can supposedly hold five to seven chunks of information. Experts with years of experience group many small ideas together in one of these chunks.[5] Thus, their seven chunks are actually much bigger. Each chunk for a novice is small. Novices often cannot assimilate understanding about a whole process with these few small chunks stored in their mind, and so they must go back multiple times to the problem statement. It becomes easier to solve the problem by finding an equation that seems appropriate and plugging the known information into that equation—the infamous plug-and-chug problem-solving strategy. Constructing a sketch of the process allows novices to see the problem situation without having to rely on storing the information in their mind. They can then focus on using a more expert-like strategy to solve the problem.[6]

In kinematics, students can use the sketch and words to construct a motion diagram. A motion diagram consists of three elements. The first element is a sequence of dots that indicate qualitatively the positions of the moving object at evenly spaced clock readings. The second element is a set of arrows that indicate the direction of motion and the relative magnitude of the object's speed. These are called $\vec{v}$ arrows. We make them relatively thin. Third, there are thicker arrows that indicate the change in velocity of an object. These are called $\Delta\vec{v}$ arrows. A $\Delta\vec{v}$ arrow in the same direction as the velocity arrows indicates that the velocity is increasing in magnitude; a $\Delta\vec{v}$ in the direction opposite to the direction of motion indicates that velocity is decreasing in magnitude. The $\Delta\vec{v}$ arrow has the same direction as the acceleration of an object. The signs of the velocity and acceleration depend on how the $\vec{v}$ and $\Delta\vec{v}$ arrows are oriented relative to a coordinate axis that is used both with the sketch and with the motion diagram. Note, for example, that the acceleration would be positive if an object were moving in the negative direction at decreasing speed. This would be difficult to understand without the help of a motion diagram. The motion diagram serves as a concrete "referent" for the kinematics quantities used to describe the process.

Students can also use kinematics graphs to represent the motion. They are probably the most difficult type of graph used in physics because they look nothing like the actual motion.[7] We prefer to use them to represent actual position–time data collected for moving objects. To use the motion diagram to construct a graph and then link the diagram and graph to each other, a student can place the motion diagram along the vertical axis to represent the position of an object and then use the horizontal axis to represent the time or clock reading. When a position-versus-time

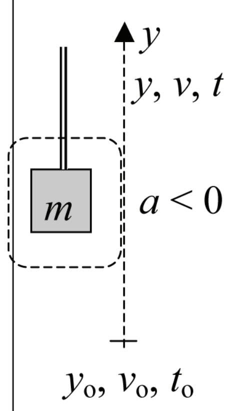
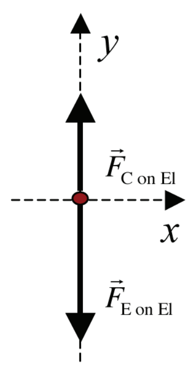
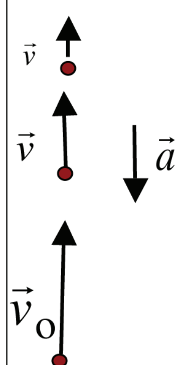
5. C. Abel, "Heuristics and Problem Solving," *New Directions for Teaching and Learning* 95 (2003): 53–58.
6. M. Chi, P. Feltovich, and R. Glaser, "Categorization and Representation of Physics Problems by Experts and Novices," *Cognitive Science* 5 (1981): 121–52; J. Larkin, J. McDermott, D. Simon, and H. Simon, "Expert and Novice Performance in Solving Physics Problems," *Science* 208 (1980): 1335–42.
7. R. Beichner, "Testing Student Interpretation of Kinematics Graph," *American Journal of Physics* 62:8 (1994): 750–62.

graph is constructed this way, it helps students avoid confusing the position of an object at every instant with the graph line. The graphs of velocity versus time can help construct kinematics equations for constant velocity and constant acceleration motion. Students can correlate the slopes of the position graphs with the directions and magnitudes of the velocity, and the slopes of the velocity graphs with the directions and magnitudes of the acceleration arrows in the motion diagram. The diagram improves understanding of the graphs.

Students can next construct kinematics equations. They check signs and values of the quantities in the equations against the motion diagrams and the graphs. Note in Figure 1 that the acceleration is positive because the car is going in the negative direction and its speed is decreasing. The reason for the sign is made clear from the motion diagram and is consistent with the slope of the velocity-versus-time graph. Students should learn to use these different representations of motion to evaluate their work; in fact, the best students do, and it is a skill that is especially helpful in solving difficult problems.[8]

Linear Dynamics Representations

FIGURE 2. A DYNAMICS PROBLEM IS REPRESENTED IN MULTIPLE WAYS

Word description	Sketch and system choice	Free-body diagram	Motion diagram	Newton's second law in component form
An elevator is slowing down on its way up.				$a_y = F_{net}/m$ $= \Sigma F_y/m$ or $a_y =$ $(F_{\text{C on El}} - F_{\text{E on El}})/m$

An example of a multiple representation description of a moderately simple one-dimensional dynamics problem is shown in Figure 2, which describes the

8. D. Rosengrant, E. Etkina, and A. Van Heuvelen, "Case Study: Students' Use of Multiple Representations in Problem Solving" (paper presented at the 2005 Physics Education Research Conference, Salt Lake City, UT), in *AIP Conference Proceedings*, Volume 818, eds. P. Heron, J. Marx, and L. McCullough (Melville, NY: American Institute of Physics, 2006), 49–52.

dynamics of an elevator car as it ascends and approaches the floor on which it will stop. Notice how we use each successive representation to support construction of the next representation. Once again, we convert the words in the problem statement to a sketch where we include much of the known information provided in the problem statement and identify the unknown quantity.

We circle lightly in the sketch a system object that is the subject of interest in this process. We then find all objects with which the system object interacts (either directly touching or without the direct contact if there is a long-range force, such as the gravitational force that the earth exerts on the object). Then we use this sketch and the identified system to construct a new representation that is called a free-body diagram. In the literature, many different ways are suggested for constructing free-body diagrams. This literature can be found in Appendix A at the end of this article. The method that we use is based on work by Heller and Reif[9] and further developed by Van Heuvelen and Etkina.[10]

We start to draw the diagram by putting a dot that represents the system object on the side of the sketch of the situation. We then use the arrows placed with their tail on the dot to indicate the interactions (the forces) of the objects outside the system with the system object. There are two very important ideas at this moment. First, the relative lengths of the force arrows should be approximately consistent with the magnitude of the forces (if the information is known). Second, all force arrows are labeled with a letter F, and there are two subscripts indicating the object that exerts the force and the system object on which the force is exerted. For example, for the force exerted by the earth on the object, we write $F_{\mathrm{E\ on\ O}}$, where E indicates the earth and O stands for the system object. We do not use such terms as the weight of an object or the tension in a rope because they might reinforce some students' pre-existing idea that forces belong to objects. This way of labeling forces later helps students identify Newton's third law of force pairs.

We suggest that students construct a motion diagram next to a free-body diagram. The direction of the velocity change arrow on the motion diagram should match the direction of the net force that emerges from the free-body diagram. This helps students develop the habit of evaluating the consistency of the diagrams and also helps decide the lengths of the force arrows. We suggest that instructors use separate qualitative reasoning activities to help students check the consistency of

9. J. Heller and F. Reif, "Prescribing Effective Human Problem-Solving Processes: Problem Description in Physics," *Cognition and Instruction* 1 (1984): 177–216.
10. A. Van Heuvelen and E. Etkina, *Active Learning Guide*, Student Edition (San Francisco: Addison Wesley Longman, 2006).

their free-body diagrams and their motion diagrams. Examples of such activities can be found in the literature.[11]

Students can next use the free-body diagram to help apply Newton's second law in component form. This helps students avoid mistakes such as calculating the force that a cable exerts on an accelerating elevator by setting the cable tension F_{cable} equal to ma_y, and neglecting the gravitational force that the earth exerts on the elevator.

Once students have learned to construct these different representations, they need to learn to use them to evaluate their work. For example, is the velocity change arrow in the motion diagram pointing in the same direction as the net force in the free-body diagram? Have all of the forces been included in the application of Newton's second law in component form? Students need to practice converting one representation to others—for example, from equations to a free-body diagram or to a word description of a problem. Examples of such activities can be found in the literature.[12]

Finally, we want to caution instructors who use the wording of Newton's second law as "$F = ma$," or in words, "Force is mass times acceleration." The wording leads students to believe that they can find *any* force by multiplying the object's mass by its acceleration. Consequently, when they read in a problem that they need to find some force and they see that the acceleration is given, they multiply the mass by acceleration to find that force. Formulating the law as "the sum of all forces exerted on an object is equal to the objects' mass times acceleration" or, even better, "the object's acceleration is directly proportional to the net force and inversely proportional to the mass of an object," helps avoid this common difficulty.

11. Van Heuvelen, "Learning to Think Like a Physicist," 891–97; Van Heuvelen, "Overview, Case Study Physics," 898–907; Van Heuvelen and Etkina, *Active Learning Guide*.
12. See citation 11.

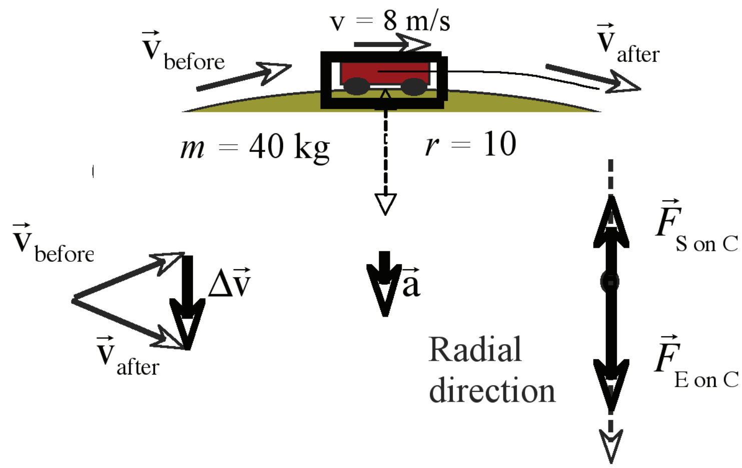
Circular Motion Dynamics

FIGURE 3. A CIRCULAR DYNAMICS PROCESS REPRESENTED IN MULTIPLE WAYS

Words: Determine the normal force that the 10-m radius hump in the road exerts on the 40-kg cart traveling at 8 m/s.

Sketch:

Diagram



Mathematics: $(40\ kg)(9.8\ m/s^2)\ -\ F_{S\ on\ C} = (40\ kg)[(8\ m/s)^2/(10\ m)]$

Figure 3 shows a multiple representation description of a circular dynamics problem. We use each successive representation to help construct the next representation. We start by converting the words in the problem statement to a sketch where we put the known information and identify the unknown.

We then circle a system object in the sketch; it is the object of interest for our analysis. Then we use this sketch and system identification to construct a free-body diagram for that system object. The free-body diagram indicates with arrows and labels all objects outside the system that interact with the system object. Again, we make a special effort to have the relative lengths of the force arrows consistent with the magnitudes of the forces, but this is not always possible. The arrows for these interactions are again labeled with subscripts indicating two objects: the object that exerts a force on the system object and the system object.

To analyze the motion qualitatively we use a two-dimensional version of a motion diagram.[13] This diagram is especially helpful if a student needs to determine the direction of the acceleration at a particular point along the circular path. We draw velocity arrows just before that point, just after the point, and tangent to the path. We place the arrows on the actual path before and after the point at which we are determining the direction of the acceleration. We make their lengths proportional to

13. Heller and Reif, "Prescribing Effective Human Problem-Solving Processes," 177–216.

the object's speed. Then we redraw these arrows tail to tail on a separate figure off to the side. A velocity change arrow $\Delta\vec{v}$ is drawn from the head of the "before" arrow to the head of the "after" arrow. The acceleration $\vec{a}$ is the ratio of the velocity change $\Delta\vec{v}$ to the time interval Δt needed for that change ($\vec{a} = \Delta\vec{v}/\Delta t$). The direction of the $\Delta\vec{v}$ arrow and consequently the direction of acceleration should be consistent with the direction of the net force in the free-body diagram. This is a very useful strategy that students can practice on separate qualitative reasoning questions in which they check the magnitudes of forces.

After students check their qualitative motion analysis and their free-body diagram for consistency, they can apply Newton's second law symbolically in component form and then evaluate the symbols. They need to apply the same reasoning strategy as in linear motion: There should be as many terms on the force side of the equation as there are force arrows in the diagram. Circular motion is an especially difficult conceptual area for students. We find it very helpful to have students represent different circular motion processes in multiple ways—so-called goal-free problems,[14] as described below.

Helping Students Learn and Apply These Representations with Formative Assessment

In the previous section we described a variety of types of representations. In this section, we provide explicit instructions for constructing and using some of the representations, including activities that can be used in the classroom to assess and modify the student learning as it progresses, if needed. Early in a unit of study, students can be asked to focus on the more qualitative representations and to use them for qualitative reasoning activities.

14. Van Heuvelen, "Learning to Think Like a Physicist," 891–97; Van Heuvelen, "Overview, Case Study Physics," 898–907.

Motion Diagrams

FIGURE 4. CONSTRUCTING A MOTION DIAGRAM

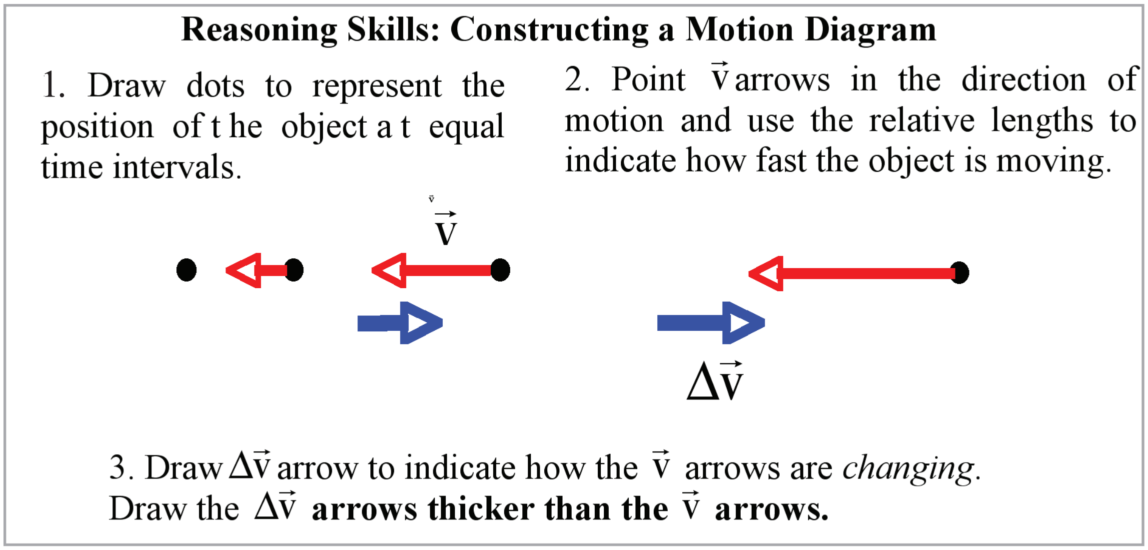


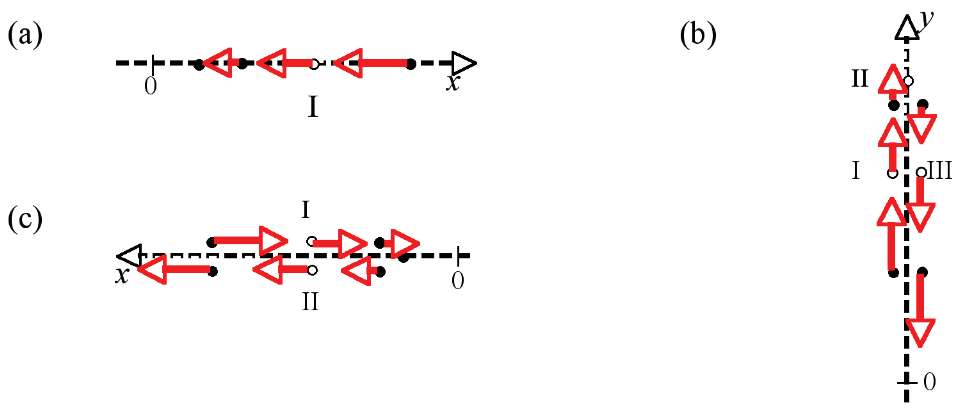
Figure 4 provides instructions for constructing a motion diagram. Students first observe a person walking slowly and dropping beanbags every second. Then they discuss how they can use the beanbags to represent the motion so that another person who did not see the motion could visualize it. Next, the instructor shows the students how to construct a formal diagram. Finally, students observe the following different types of motion and construct motion diagrams for different parts of the motion using the sequence of activities found in the literature:[15]

- a dynamics cart moving at constant speed on track;
- the cart being pushed gently so it moves faster and faster;
- the cart pushed gently opposite its velocity so that it moves slower and slower;
- a ball thrown vertically upward, as it is on the way up;
- a ball thrown vertically upward, as it is on the way down; and
- a ball thrown vertically upward, just before reaching the top of its vertical trajectory and just after (to see that the acceleration is down for the entire trip).

15. Van Heuvelen and Etkina, *Active Learning Guide*, 2–17.

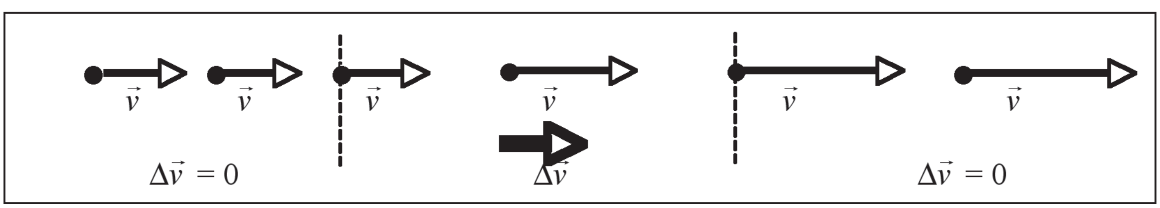
Figure 5. Signs of *x*, *v*, and *a* relative to the coordinate axis. The answers in the order (x, v, a): (a) (+, -, +), (b) I (+, +, -), II (+, 0, -), III (+, -, -), (c) I (+, -, -), II (+, +, +)

Represent and reason: The motion diagrams in the illustrations represent the motion of different objects modeled as particles. The arrows are velocity arrows.



A different coordinate axis is provided in each situation. An open circle (see Roman numerals) indicates the locations of interest. Draw acceleration arrows on each diagram. Determine the signs of the position, velocity, and acceleration relative to the coordinate axis for the open circle position.

Figure 6. Invent a real process consistent with the motion diagram. The dashed lines separate different parts of the motion. One possible answer (there are many). Part 1: Car 1 is moving at constant velocity behind car 2. Part 2: Car 1 speeds up while passing car 2. Part 3: Car 1 then travels at a faster constant velocity.



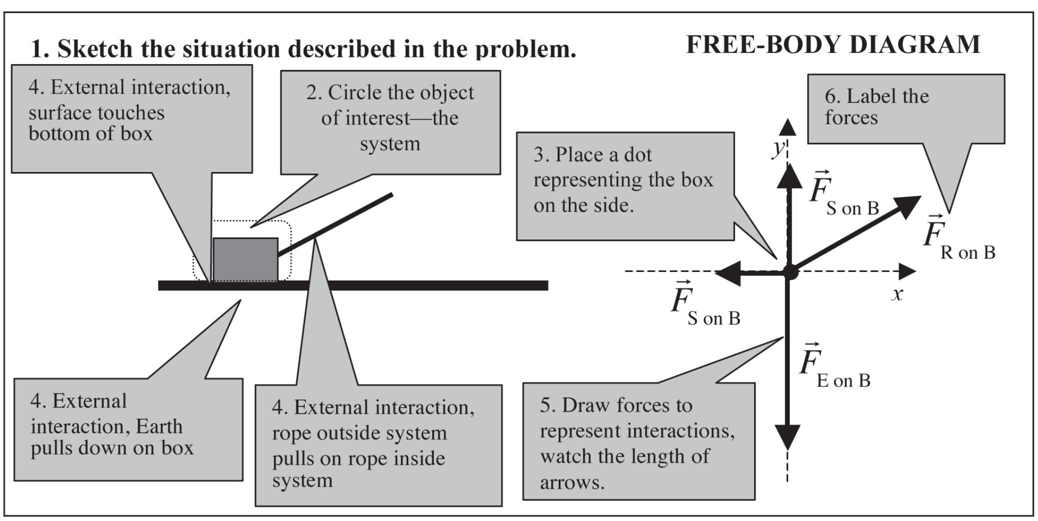
Another useful activity is to ask students to determine the signs of the moving object's position, velocity, and velocity change relative to the coordinate axis at the positions of the five open dots in the motion diagrams in Figure 5. They can also be asked to describe motion depicted by a given motion diagram, as in Figure 6.

For formative assessment activities, you can give students a motion diagram for a complicated motion (for example, an object moves at a constant velocity and then slows down to a stop) and ask them to devise a story about this motion. Another approach is to give a list of short descriptions of different motions and a list of motion diagrams, and then ask students to match them. Finally, you can ask students to draw a motion diagram for an object moving to the left and slowing down, and then moving to the right and speeding up. Then you can ask whether the direction of the $\Delta\vec{v}$ arrows on the diagram indicates whether an object is speeding up or slowing down. After students learn graphs, formative assessment activities can involve moving from one representation to another, especially from velocity-versus-time graphs to motion diagrams and back.

Free-Body Diagrams

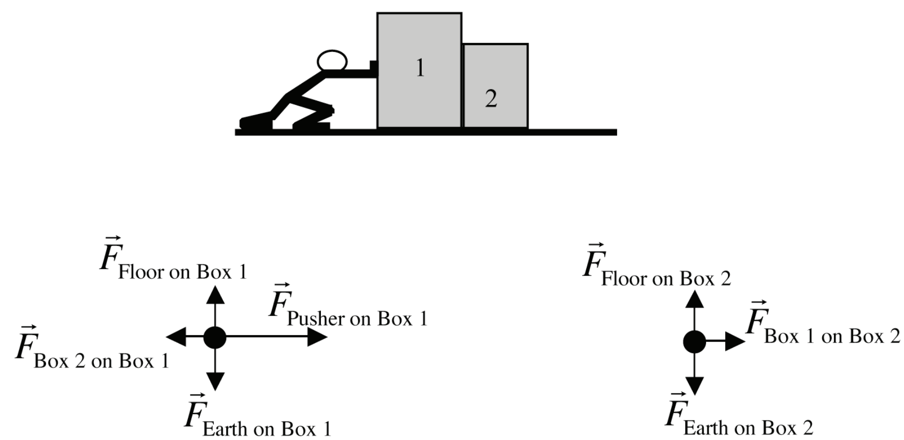
To learn to draw free-body diagrams, students can start by holding a tennis ball and a medicine ball. Then they need to identify the objects that interact with each ball and represent the interactions of the balls with arrows. At this point, students often express the idea that the air is one of the objects interacting with the balls and that it pushes down on the ball. One can test this idea by having an object attached to a spring, first in the air and then under a vacuum jar. (For a video of this experiment, go to http://paer.rutgers.edu/pt3/movies/bottle_in_vacuum.mov.) After students realize that the air has a negligible effect, students can proceed to represent the interaction of the balls with the earth (here it is important to identify the earth as a pulling object and not a mysterious "gravity" that students like so much) and the hand. The lengths of the arrows for the two balls represent different efforts that students have to exert to keep the medicine ball steady compared to the tennis ball. After students draw the arrows, the teacher can tell them to redraw the picture with a dot instead of the object of interest and explain how to label the arrows. At this point, students can be given explicit instructions for constructing free-body diagrams (see Figure 7).

FIGURE 7. CONSTRUCTING A FREE-BODY DIAGRAM



Having learned the method, students can be asked to consider two books sitting on a tabletop, with one book on top of the other. They should draw a free-body diagram of each book. Have them start with the actual books on a table, then ask them to draw a sketch of the situation and circle one book that is the object of interest. Drawing the circle is not trivial—it should carefully pass between the contacting surface of the system object and an outside touching object in the environment. The arrows shown in the diagram should be due to forces exerted by an object outside the system that is touching the object on the inside. For example, for the book on the table, students should show one upward arrow due to the force exerted by the table on the bottom book ($F_{\text{T on BB}}$), and two downward forces due to the force exerted by the top book on the bottom book ($F_{\text{TB on BB}}$) and the gravitational force exerted by the earth on the bottom book ($F_{\text{E on BB}}$).

Figure 8. Construct separate free-body diagrams for each block. Ignore friction between blocks and the surface.



After students have completed the above steps, have them construct a free-body diagram for each block shown in Figure 8 (ignore friction). Block 1 exerts a force on block 2. The person is not touching block 2 and does not exert a force on it.

An instructor can use free-body diagrams and motion diagrams together to help students understand the relationship between an unbalanced force and the *changes* in motion of an object, not the motion itself. Some useful experiments for analysis here are motion diagrams and free-body diagrams for a low-friction cart on a track that is:

- first being pushed gently by a person so it moves faster and faster;
- then moving at constant velocity with no pushing; and
- then being pushed gently opposite its motion so its speed decreases.

For formative assessment, you can ask students to predict the change in the reading of a scale that is lifting an object from rest and then slowing to a stop. For a successful analysis of this situation, students need to draw a motion diagram, a free-body diagram, and a velocity–time graph. To test their reasoning, they can view the video of the scenario at http://paer.rutgers.edu/pt3/experiment.php?topicid=3&exptid=172.

To help students construct and apply the idea that there is no need to draw an extra force in the direction of motion, you can use a projectile. Ask students to construct a free-body diagram for a projectile at different places along its path. They often include force arrows in the direction of motion but cannot identify the other object exerting the force. As they remember that the word "force" denotes an interaction between two objects, students should understand that there is no force in the direction of motion because they cannot identify another object pushing or pulling the projectile in that direction.

Velocity Subtraction Acceleration Diagrams

Figure 9. A method to estimate the direction of acceleration during two-dimensional motion.

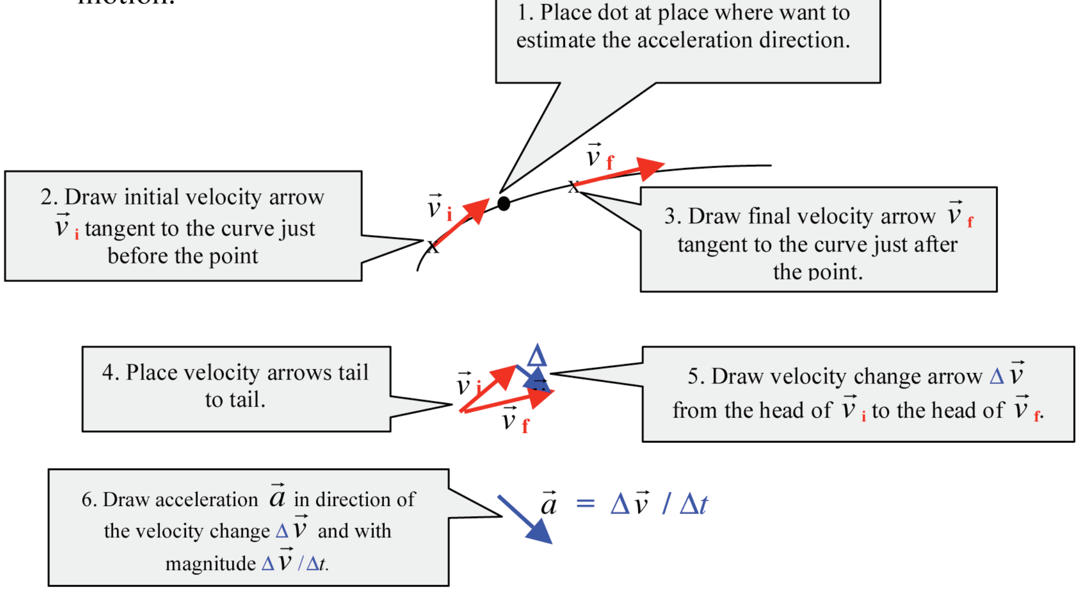
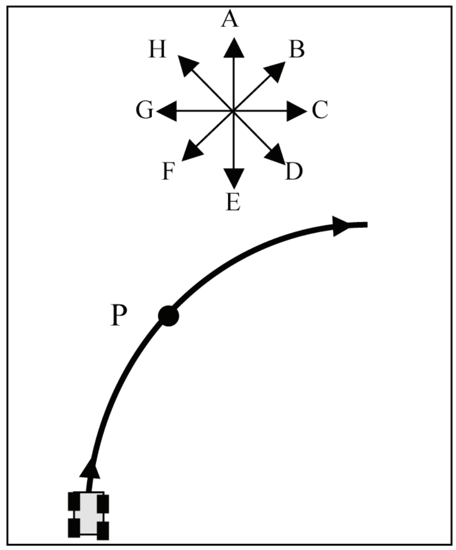


Figure 9 provides instructions for using a diagrammatic velocity subtraction method to estimate the acceleration direction during two-dimensional motion.[16] These diagrams are very useful in helping students understand that there is acceleration during circular motion of constant speed. Note that students tend to draw such diagrams with very short arrows, making them difficult to use and understand. Some draw the velocity change arrow from the head of the final velocity to the head of the

16. Heller and Reif, "Prescribing Effective Human Problem-Solving Processes," 177–216.

initial velocity, which is the opposite of the direction in which they should be drawn. For practice they can be given an activity such as that shown in Figure 10.[17]

Figure 10. Which arrow is closest to the acceleration direction at point P when: (a) the car is moving at constant speed (D); (b) the car's speed is continually increasing (C); and (c) the car's speed is continually decreasing (E)?

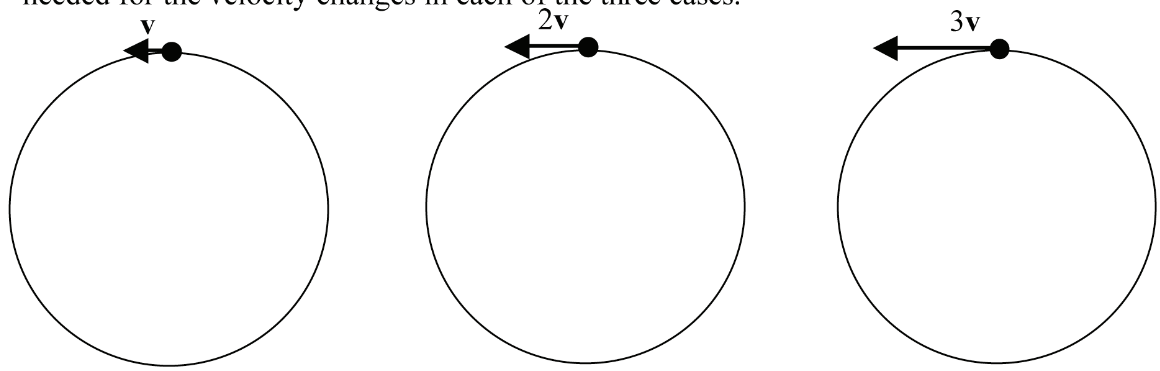


We use these diagrams not only to determine the direction of the acceleration during curvilinear motion as we discussed above, but also to help determine the magnitude of the centripetal acceleration—to see the v^2 dependence (see Figure 11) and the $1/r$ dependence (see Figure 12). This is a concrete way to arrive at the conclusion that $a_c = v^2/r$.

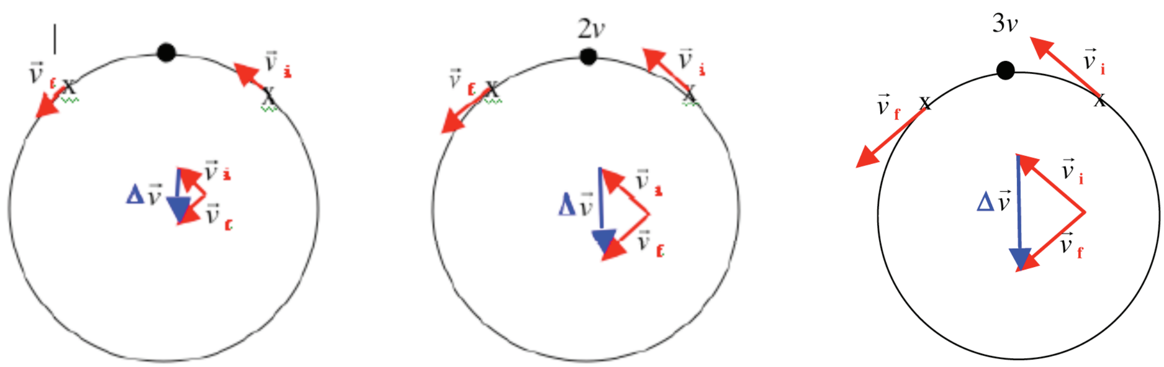
17. D. T. Brookes, "The Role of Language in Learning Physics" (Unpublished Ph.D. dissertation, Rutgers, The State University of New Jersey, 2006), 190.

Figure 11. What is the dependence of the centripetal acceleration of an object's speed during constant speed circular motion?

Reason: Three small toy cars travel at constant speed in identical-radii horizontal circular paths. Car *A* moves at speed *v*, car *B* at speed 2*v*, and car *C* at speed 3*v*. Use the graphical velocity subtraction method to determine how the magnitude of the acceleration of the cars depends on their speeds. Remember that acceleration is $\Delta\vec{v}/\Delta t$ and that you need to compare the velocity change $\Delta\vec{v}$ vectors for the three speeds and also the time interval Δt needed for the velocity changes in each of the three cases.



Solution:

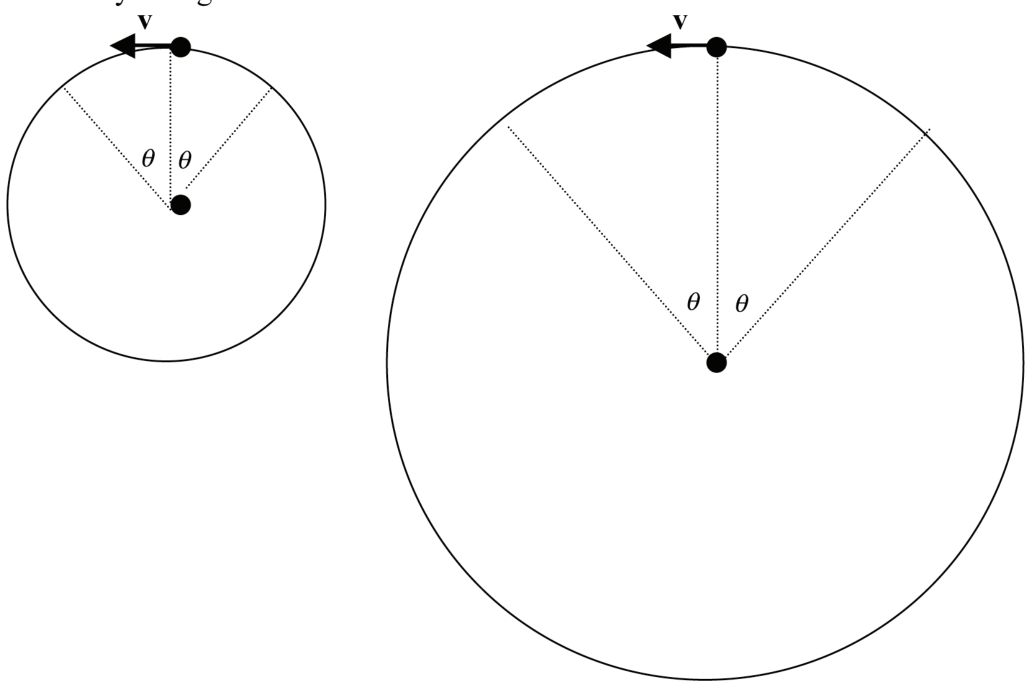


$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} \qquad \vec{a} = \frac{2\Delta\vec{v}}{\Delta t/2} = 4\frac{\Delta\vec{v}}{\Delta t} \qquad \vec{a} = \frac{3\Delta\vec{v}}{\Delta t/3} = 9\frac{\Delta\vec{v}}{\Delta t}$$

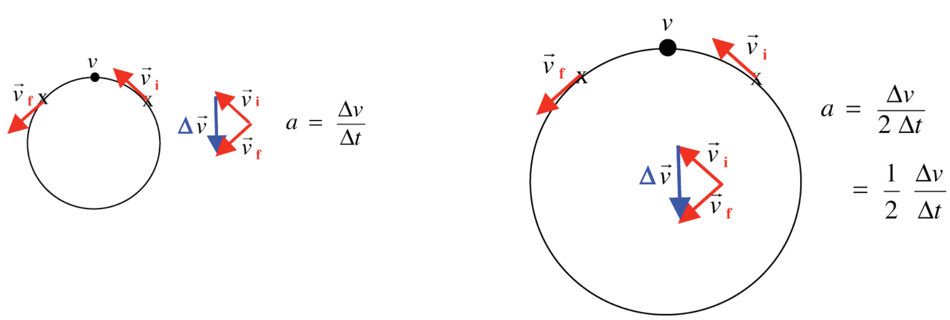
$$a \,\alpha\, v^2$$

Figure 12. What is the dependence of the centripetal acceleration of the radius of the circle during constant speed circular motion?

Reason: Two small toy cars travel at the same constant speed in horizontal circular paths. Car I moves in a circle of radius *r* and car II in a circle of radius 2*r*. (a) Use the graphical velocity change method to determine how the magnitude of the acceleration of the cars depends on the radii of the circles. Do not forget to consider the time intervals needed for the velocity changes.



Solution:



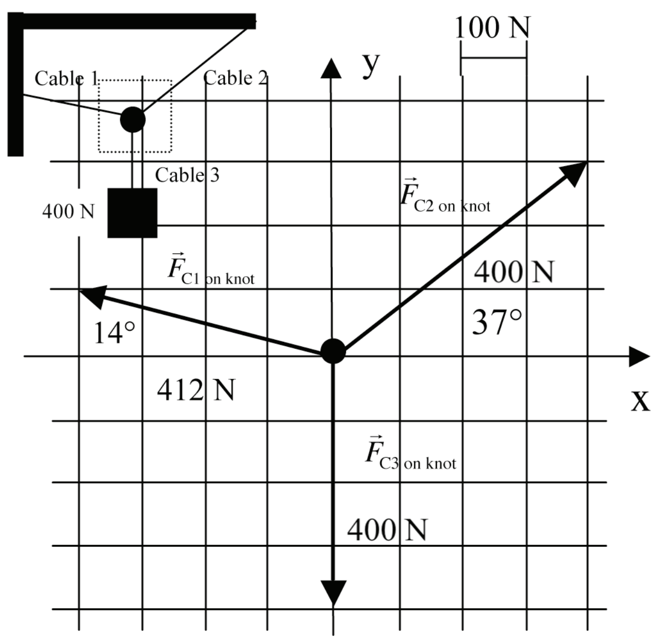
$$A \;\alpha\; \frac{1}{r}$$

A good formative assessment combines the velocity subtraction technique and the free-body diagram. This could be a question that arises when students need to construct a free-body diagram for a pendulum bob as it passes the lowest point of its swing. After students determine the direction of the acceleration and then adjust the length of the arrows, they can test their reasoning using a videotaped experiment such as the one at http://paer.rutgers.edu/pt3/experiment.php?topicid=5&exptid=59. If you do this activity with the students, you will notice again that some of them include a force in the direction of motion. Ask the students to identify the object causing this force. They might call it the force of motion, but they might not find another object causing the force. If they cannot find an object that interacts with the pendulum bob to create a force, the force has to be removed from the diagram.

Research indicates that this problem is very difficult for the students. In one survey, about 60 percent of engineering students at the end of their study included a horizontal arrow in the direction of motion;[18] however, in a course where the above methods were used, fewer than 5 percent of the students made the same mistake.[19]

Using Free-Body Diagrams to Apply Newton's Second Law in Component Form

Figure 13. Visualizing the *x* and *y* components of the forces that ropes exert on a knot

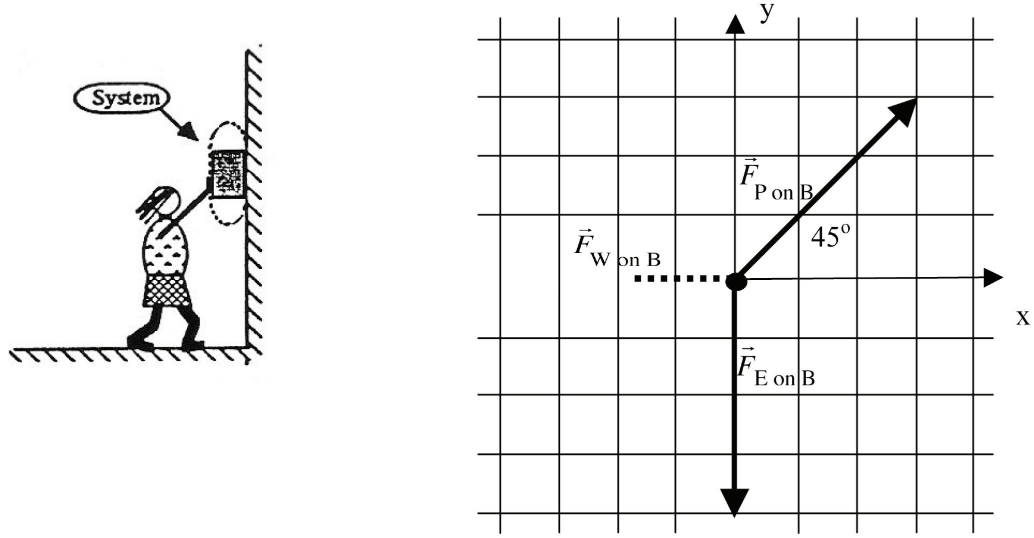


Free-body diagrams help students write Newton's second law in component form. Figure 13 describes a situation in which the component form of analysis is required. Students visualize the components by drawing their projections on axes and then use the component equations to confirm the results mathematically. Then they can work on an assignment where they need to use a sketch together with a partially

18. Van Heuvelen, "Overview, Case Study Physics," 898–907.
19. Brookes, "The Role of Language in Learning Physics," 41.

completed free-body diagram to answer questions about unknown forces. Students can check their work using Newton's second law in component form (Figure 14).

Figure 14. (a) By inspection determine the magnitude of the normal force that the wall exerts on the box so that the box does not accelerate horizontally. (b) By inspection determine the net force in the vertical direction. (c) Apply Newton's second law in component form and check your results for (a) and (b) mathematically.



A formative assessment might give the application of Newton's second law in component form and then ask a student to construct a free-body diagram that is consistent with equations. The assessment might also ask the student to write a problem that could be solved with the equations. This assessment is sometimes called Equation Jeopardy.[20] An example of component equations is provided below:[21]

x: + (100 N) cos 37° + 0 – 0.40 N + 0 = (10 kg) a_x

y: + (100 N) sin 37° + $F_{\text{ground on object}}$ + 0 – (10 kg)(10 N/kg) = (10 kg) 0

Jeopardy problems are very valuable because they have multiple correct solutions and thus help students develop epistemic cognition.

Goal-Free Problems

After students have learned to draw and use these different representations, you can now give them any end-of-chapter problem of medium difficulty and ask them to represent it in different ways without solving the problem. The situation is described first in words. The students then make a sketch that has coordinate axes, and they include the known information and identify the desired unknown. They make a motion diagram or velocity subtraction diagram to determine the acceleration

20. A. Van Heuvelen and D. Maloney, "Playing Physics Jeopardy," *American Journal of Physics* 67 (1999): 252–56.
21. Van Heuvelen and Etkina, *Active Learning Guide,* 3-1.

direction, they construct a free-body diagram, and they apply Newton's second law in component form and apply kinematics equations if needed. These different representations should be checked for consistency. Students who have spent time doing such activities have significantly improved scores on problem-solving tests compared to students who do not use multiple representations.[22]

Does This Strategy Work as Measured by Summative Assessment Tools?

Many educators suggest that multiple representations play a positive role in student learning (see references on problem solving in Appendix A). In the early 1990s Van Heuvelen developed an Overview, Case Study Physics (OCS) curriculum that emphasized multiple representations.[23] The curriculum was adapted from a very successful high school Overview, Case Study physics instruction system developed by Art Farmer at Gunn High School in Palo Alto, California.[24]

The college OCS version has been used at several colleges and universities, with results reported on various standardized conceptual and problem-solving tests. Van Heuvelen compared students' learning gains on a diagnostic test from the reformed course (in which the instructor created a representation-rich environment) and a traditionally taught course and found that student gains in the reformed course were 15 percent higher than those in the traditional class. He also discovered that students were able to retain information longer. Gautreau used the method at New Jersey Institute of Technology.[25] His students' scores were dramatically higher than students' scores in three traditionally taught classes. The methodology was persuasive: The three professors teaching the traditional classes wrote the tests, the tests were not seen by Gautreau before they were administered, and department graduate students did the grading on all of the tests.

De Leone and Gire studied how many representations students in a reformed course used when solving open-ended problems on quizzes and tests.[26] They found that students who correctly solved the majority of the problems tended to use multiple representations frequently.

22. R. Gautreau and L. Novemsky, "Concepts First—A Small Group Approach to Physics Learning," *American Journal of Physics* 65 (1997): 418–28.
23. Van Heuvelen, "Overview, Case Study Physics," 898–907; A. Van Heuvelen, "Millikan Lecture 1999: The Workplace, Student Minds, and Physics Learning Systems," *American Journal of Physics* 69 (2001): 1139–46.
24. Arthur V. Farmer, "A New Approach to Physics Teaching," *The Physics Teacher* 23 (1985): 338–43.
25. Gautreau and Novemsky, "Concepts First," 418–28.
26. C. DeLeone and E. Gire, "Is Instructional Emphasis on the Use of Non-Mathematical Representations Worth the Effort?" (paper presented at the 2005 Physics Education Research Conference, Salt Lake City, UT), in *AIP Conference Proceedings*, Volume 818, eds. P. Heron, J. Marx, and L. McCullough (Melville, NY: American Institute of Physics, 2005), 45–48.

From 2003 to 2006 the authors of this article conducted a project in two large-enrollment (about 500 students in one and 200 in the other) algebra-based physics courses for science majors at Rutgers University in which they used all of the strategies described above, integrated in the Investigative Science Learning Environment (ISLE) format.[27] The goal of the study was to find whether students in a course that incorporated all the multiple-representation strategies described above (and many more) actually used these representations, specifically free-body diagrams to solve problems in mechanics and electrostatics.

The course was comparable to high school honors physics, and sequences of students' activities used in the course were taken from *The Physics Active Learning Guide* (*ALG*).[28] A multiple-representation approach was emphasized in a coordinated way in lectures, recitations, and laboratories.

As the course had such high enrollment, the exams were mostly multiple choice and students received no credit for the work on multiple-choice questions. Even though students received no credit for their work on multiple-choice exam problems (which composed about 80 percent of the exams), students had drawn free-body diagrams in 66 percent of the solutions for seven multiple-choice exam problems that involved forces, even when solving problems in mechanics and electrostatics. We also found that student grades correlated highly with their choice of whether to draw a free-body diagram to solve a problem. "A" students used free-body diagrams 84 percent of the time, "B+" and "B" students used them 71 percent of the time, and those students who had a C+ or lower only used them 45 percent of the time.

Table 1

Rubric for Coding Free-Body Diagrams

0-no evidence	1- inadequate	2-needs improvement	3- adequate
No representation is constructed.	FBD is constructed but contains major errors such as missing or extra forces. Forces may be pointed in the wrong direction.	FBD contains no errors such as missing or extra forces but lacks a key feature such as labels or forces are mislabeled or do not contain a labeled axis if appropriate. Lengths of force arrows could be incorrect.	The diagram contains no errors and each force is labeled so that it is clear what each force represents.

27. E. Etkina and A. Van Heuvelen, "Investigative Science Learning Environment: Using the Processes of Science and Cognitive Strategies to Learn Physics," in *Proceedings of the 2001 Physics Education Research Conference*, eds. S. Franklin, J. Marx, and K. Cummings (Rochester, NY: American Institute of Physics, 2001), 17–21.
28. Van Heuvelen and Etkina, *Active Learning Guide*.

We also developed a rubric to evaluate the quality of student free-body diagrams (see Table 1) and used this rubric to find whether those students who drew better diagrams on their exam sheets were more successful in solving the problem. We found that this was indeed the case. We analyzed 1,465 student solutions for 12 different problems. We found that 85 percent of the students whose free-body diagram score was a 3 solved the problem correctly, 71 percent of those whose diagrams received a score of 2 solved the problem correctly, and only 38 percent of those whose diagrams received a score of 1 (basically incorrect diagrams) solved the problem correctly. Forty-nine percent of students who did not draw a diagram solved the problem correctly. These latter students could have constructed diagrams in their head, as was stated by a student during the qualitative study, or possibly on scrap paper that was not turned in to the proctors, as was stated by some students in follow-up interview studies. The difference among these groups is statistically significant, and the details of the analysis are described in the literature.[29] Clearly, correctly drawn free-body diagrams helped students solve the problems. Remember, the students drew the diagrams despite the fact that they received no credit for this work (they were answering multiple-choice problems).

Figure 15. Electrostatics question used during think-aloud studies with students.

Question 1: A small metal ball with +2.0 μC of charge hangs at the end of a vertical string. A second identical ball with -2.0 μC of charge hangs at the end of a second vertical string. The tops of the strings are brought near each other and the strings reach an equilibrium orientation (no longer vertical) when the balls are 3.0 cm apart. If the force exerted by the Earth on each ball is 30 N, what is the force exerted by the string on the ball?

In addition, we videotaped several students from these courses while working on a static electricity problem and for which a free-body diagram would be helpful (see Figure 15). However, the text of the problem did not ask students to draw a diagram. Six students (two high achieving, two medium achieving, and two low achieving) solved the problem using "a think-aloud protocol" during which they had to speak while solving the problem so an observer could record their thinking.[30] We found that the best students not only drew free-body diagrams to write a mathematical description of the problem, but they also repeatedly returned to the

29. D. Rosengrant, "Multiple Representations and Free Body Diagrams: Do Students Benefit from Using Them?" (Unpublished Ph.D. dissertation, Rutgers, The State University of New Jersey, 2007), 35.
30. K. Ericsson and H. Simon, *Protocol Analysis: Verbal Reports as Data* (Cambridge, MA: MIT Press, 1984); K. Ericsson and H. Simon, "How to Study Thinking in Everyday Life: Contrasting Think-Aloud Protocols with Descriptions and Explanations of Thinking," *Mind, Culture, and Activity* 5:3 (1998): 178–86.

diagram during the solution process. Medium-achieving students also drew free-body diagrams and used them to write mathematical equations but did not use the diagram for evaluation purposes. Low-achieving students either did not draw a diagram or drew it in addition to the equations, but not to help construct them. The details of the study can be found in the literature.[31]

Summary and Implications for Instruction

Multiple representations are the tools that scientists use to construct new knowledge, solve problems, evaluate their work, and communicate. If we want our students to reason like scientists, we need to engage them in similar activities and convince them of the usefulness of representations. Summative assessment is one way to communicate this idea. If students are asked on an exam to represent a situation in multiple ways without solving for a particular quantity, they will understand that the ability to re-express concepts has value. Another way to achieve the same goal is to provide students with problems that are difficult to solve without representing the situation in multiple ways.

In this paper we presented an approach for helping students represent kinematics and dynamics processes in multiple ways. We shared data that indicate that students using these methods are more successful. Furthermore, on certain examination questions those students who are most successful in the classroom evaluate their work by using the different representations.

31. Rosengrant, Etkina, and Van Heuvelen, "Case Study: Students' Use of Multiple Representations in Problem Solving," 49–52.

Appendix A

Additional literature on translating between representations:

A. diSessa and B. Sherin, "Meta-Representation: An Introduction," *Journal of Mathematical Behavior* 19 (2000): 385–98.

R. Dufresne, W. Gerace, and W. Leonard, "Solving Physics Problems with Multiple Representations," *The Physics Teacher* 35 (1997): 270–75.

G. Goldin, "Representation in School Mathematics: A Unifying Research Perspective," in *A Research Companion to Principles and Standards for School Mathematics*, eds. J. Kilpatrick, W. G. Martin, and D. Schifter (Reston, VA: National Council of Teachers of Mathematics, 2003), 275–85.

Heller and Reif, "Prescribing Effective Human Problem Solving Processes," 177–216.

D. Hestenes, "Toward a Modeling Theory of Physics Instruction," *American Journal of Physics* 55 (1987): 440–54.

J. Larkin, "Understanding, Problem Representations, and Skill in Physics," *Thinking and Learning Skills* 2 (1985): 141–59.

Larkin and Simon, "Why a Diagram is (Sometimes) Worth Ten Thousand Words," 65–99.

Additional literature on representations involving a free-body diagram:

K. Fisher, "Exercises in Drawing and Utilizing Free-Body Diagrams," *The Physics Teacher* 37 (1999): 434–35.

B. Lane, "Why Can't Physicists Draw FBDs?" *The Physics Teacher* 31 (1993): 216–17.

D. Maloney, “Forces as Interactions,” *The Physics Teacher* 28 (1990): 386–90.

M. Mattson, “Getting Students to Provide Direction When Drawing Free-Body Diagrams,” *The Physics Teacher* 42 (2004): 398–99.

R. Newburgh, “Force Diagrams: How? and Why?” *The Physics Teacher* 32 (1994): 352.

A. Puri, “The Art of Free-Body Diagrams,” *Physics Educator* 31 (1996): 155–57.

Additional literature on the use of multiple representations in problem solving:

Dufresne, Gerace, and Leonard, “Solving Physics Problems with Multiple Representations,” 270–75.

Hestenes, “Toward a Modeling Theory of Physics Instruction,” 440–54.

Larkin, “Understanding, Problem Representations, and Skill in Physics,” 141–59.

Rosengrant, Etkina, and Van Heuvelen, “Case Study: Students’ Use of Multiple Representations in Problem Solving,” 49–52.

Van Heuvelen, “Learning to Think Like a Physicist,” 891–97.

Using Multiple Representations to Understand Energy

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Introduction

Energy is a very abstract concept. The full importance of energy was not recognized until Joule's experiments of the mid-nineteenth century, nearly 200 years after Newton's flash of genius. And unlike the well-defined idea of momentum, $p = mv$, we keep "inventing" new forms of energy—kinetic energy, potential energy, thermal energy, nuclear energy, and so on. Energy, at least to beginning students, is an amorphous, ill-defined concept. It's not at all obvious how $\frac{1}{2}mv^2$ has any connection to thermal energy or nuclear energy. And students are certainly not helped by our everyday use of the terms "work" and "energy." Why, after all, should we be worried about "conserving energy" if energy is always conserved?

One difficulty is that there's no *thing* you can put your finger on and say, "Here, this is energy." It's just some number—calculated by adding a little of this and a little of that—that for some hard-to-fathom reason never changes. In addition, energy as it's used in thermodynamics seems to have little connection to energy as it had been defined in mechanics. We tend to say that the first law of thermodynamics is a "general statement of energy conservation," but students rarely see it that way.

To compound the difficulties, the presentation of energy to students is quite spread out. In a class situation, thermodynamics usually is presented weeks after energy had been introduced in the context of mechanics, and students will already have forgotten some of what they learned earlier.

Given a mechanics problem with no problem-solving hints, most students will choose a Newton's law approach even where an experienced physicist or physics teacher would find an energy solution to be much simpler. When I've asked students about this after exams, noting how much extra effort they spent solving (or, often, failing to solve) a problem that would have been "easy" with an energy approach, their response has been that forces and motion laws are tangible and concrete, whereas energy seems mysterious, and they have no confidence in their ability to use energy laws correctly.

There are similar difficulties with heat and heat engines in thermodynamics. Students may become reasonably adept at calculating the work done in a cycle or the efficiency of an engine—plug-and-chug tasks—but few can explain in words what a heat engine is doing as it transforms heat energy into mechanical energy.

Goals

There is a large-scale coherence to the "energy story" that is missing from most introductory presentations but that is essential for a firm grasp of the concept. The overall goal is to help students see the big picture of what energy is all about. In particular:

1. To understand how the concept of energy is used in mechanics.
2. To understand how the concept of energy is used in thermodynamics.
3. To recognize that the first law of thermodynamics—and thus the use of energy in thermodynamics—is closely related to and a continuation of the energy ideas introduced in mechanics.

Energy is a big topic, and this essay cannot possibly address all the many facets of energy. The focus of this essay is on using multiple representations to better understand energy. Many other vital issues—types of energy, energy transformations, work and heat, heat engines, etc.—must also be part of the lesson plan but will be mentioned here only peripherally, if at all.

Because the teaching of energy is spread out in time, the information presented in this essay does not form the basis for a lesson plan that can be presented on sequential days or even sequential weeks. The challenge to the teacher is twofold. First, to make sure *you* understand the ideas well enough to lay out a coherent teaching plan extending over many weeks. Second, upon reaching thermodynamics, to keep linking back to mechanics, providing the storyline and the big picture that are woefully absent in most textbooks. The numerical examples in this essay are intended to give you ideas for appropriate class examples or homework problems.

Multiple Representations

We represent knowledge in many ways and forms. The words with which a physics problem are written demonstrate a *verbal representation* of knowledge, as do the words with which a student expresses his or her understanding. The equations with which the problem is solved are *mathematical representations* of the same knowledge. Unfortunately, most students try to leap directly from a problem statement to a mathematical solution. In other words, they follow a plug-and-chug problem-solving strategy in which they try to "pattern match" the numbers in the problem to an equation in the book, then cross their fingers and hope it's the right equation.

Few physics problems—and certainly not the types of problems we expect students to solve to demonstrate mastery—can be solved with such a simple approach. Students must learn to *reason* with physics concepts, and that's where other representations of knowledge are important. An obvious one, the *pictorial representation*, is a picture of the situation. But it is not just any picture. A useful picture, one that's going to help solve the problem, follows procedures that teachers must teach and students must learn. Simply telling students to draw a picture is not sufficient.

Other representations of knowledge include graphical representations, bar charts, interaction diagrams, and energy transfer diagrams. Examples will be given below. The point is that much of *learning* physics consists of learning how to move back and forth between different representations of knowledge. Think of these representations as different perspectives on the same situation. The more perspectives you can bring to bear on a problem or a situation, the more likely you are to understand what's going on and then to solve the problem. Experts move back and forth between representations without conscious effort, but it's a skill students must learn and thus a skill teachers must be comfortable teaching.

Lesson 1: Conservation of Mechanical Energy

Objective: Mechanical energy is conserved in an isolated system.
Representations: Before-and-after pictorial representations, bar charts, interaction diagrams, energy graphs.
The typical textbook starts the first chapter on energy with "Define work as..." This is an abstract, top-down approach with no rationale or motivation. It has long been recognized that students learn better with a concrete, bottom-up approach based on their experience and on simple demonstrations and experiments. Only *after* students

grasp some initial ideas about energy does it make sense to introduce "work" as a way in which a system exchanges energy with its environment.

In my textbooks, I start the energy chapter by using Newton's laws and simple kinematics to "discover" that the quantity $\frac{1}{2}mv^2 + mgy$ doesn't change for an object on a frictionless incline. Similarly, the quantity $\frac{1}{2}mv^2 + \frac{1}{2}kx^2$ doesn't change for an object on a spring. Notice two important things:

1. Each quantity is a sum of two terms. One term depends entirely on speed, regardless of where the object is. The other depends entirely on position, regardless of speed. Let's call these "kinetic energy" and "potential energy," following the dictum "idea first, name second." That is, we've quickly found evidence that the quantities $\frac{1}{2}mv^2$ and mgy may be useful, so let's give them names to make it easier to talk about them. (Contrast this with the abstract approach of starting with definitions that have no rationale.) We can then go on to say that their sum—the quantity that doesn't change—is called "mechanical energy."
2. Because the mechanical energy is conserved—at least in the situations looked at thus far—it has the same value *before* an interaction (a gravitational interaction or an elastic interaction) as *after* the interaction. This is the basis for the first important visual representation.

Note: Even if you are using a textbook that follows the traditional "Define work as..." approach, I urge you to start with some simple ideas about kinetic and potential energy in order to provide a context for this definition of work.

We are always telling students to "draw a picture," but it is not obvious to students *what* they should draw or why this is useful. I remind students that they can hold only five or six pieces of information in their short-term memory, so by the time they finish reading a typical problem, they are already forgetting how it started! Thus, one purpose of a picture is to be a "brain extender" that helps to keep track of information. A second purpose is to help structure a solution to the problem. However, what students need to draw depends on the nature of the problem. A drawing for an energy conservation problem differs from the drawing for a Newton's law problem.

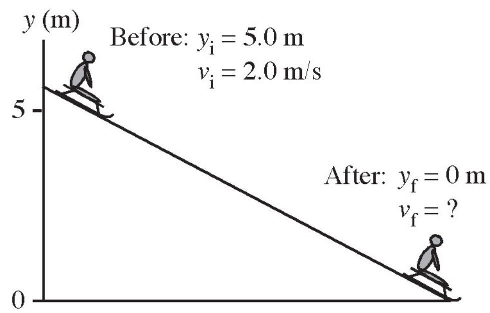
The important idea behind conservation law problems is that *the details of the interaction do not matter*. (This is very different from Newton's laws, which are focused on the interaction.) Consider the following typical problem:

> Christine runs forward with her sled at 2.0 m/s. She hops onto the sled at the top of a 5.0 m high, very slippery, snow-covered slope. What is her speed at the bottom?

We can use energy to relate the situation before she slides to the situation at the bottom of the hill. The details of the sliding—assumed to be frictionless—are irrelevant. Thus the Figure 1 *before-and-after pictorial representation* (1) establishes a coordinate system; (2) captures all the essential information; and (3) identifies what we're trying to find, in this case the final speed, v_f. Now we can read the pertinent information right off the picture and use it in the mathematical representation of the problem—namely, the energy conservation equation:

$$\tfrac{1}{2}mv_\mathrm{i}^2 + mgy_\mathrm{i} = \tfrac{1}{2}mv_\mathrm{f}^2 + mgy_\mathrm{f}$$

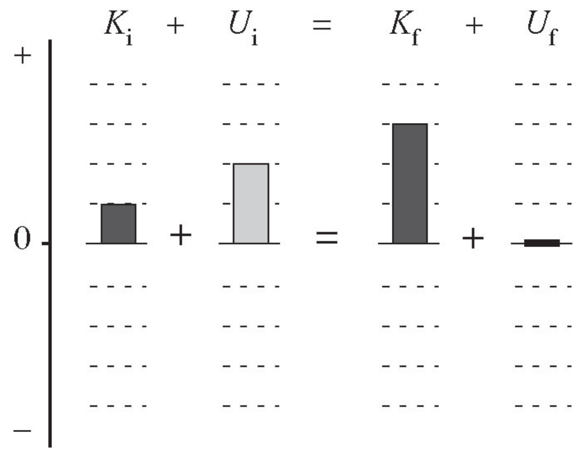
FIGURE 1: A BEFORE-AND-AFTER PICTORIAL REPRESENTATION



This is a pretty simple problem, but it is important to get students in the habit of drawing pictures like this *before* they get to more complex problems. Then they'll have a useful problem-solving tool. If you delay introducing tools until the problems become difficult, students will already have developed poor problem-solving techniques and are unlikely to change. The key idea here is that we moved information from a verbal representation (the problem statement) to a pictorial representation and then to a mathematical representation (the pertinent equation). The before-and-after representation can be applied to any problem with a conserved quantity, including momentum problems and problems of charged particles in electric fields.

Recognizing that the total energy doesn't change leads to another representation of energy knowledge: *bar charts*. These are accounting devices much like those used to indicate income and expenditures. Figure 2 is a bar chart representation for the above problem of Christine on her sled. Here we see that potential energy decreases and kinetic energy increases—a *transformation* of energy from one form to another.

FIGURE 2: A BAR CHART REPRESENTATION



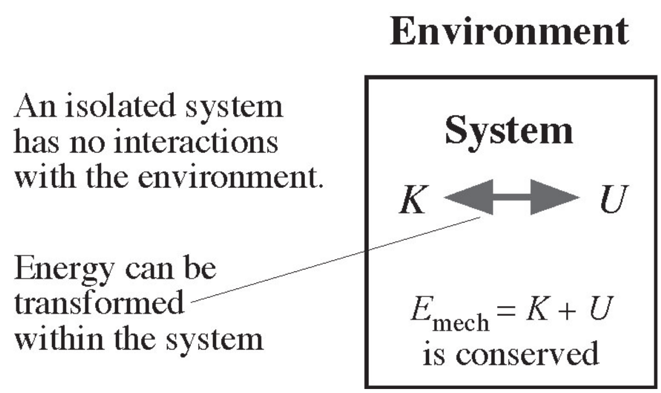
Although before-and-after pictorial representations can easily be drawn freehand, it's best to provide blank bar charts with grids if you want students to draw them accurately. The purpose of bar charts is to show (1) how energy is being transformed; and (2) that the total energy is conserved. Numerical values are not important—bar heights are chosen to be plausible, but students can differ in their choices—and in fact it's best to have students do these when analyzing a problem statement *before* getting to the mathematics. Notice that the chart allows for negative energies, although they don't occur in this example.

Note: Bar charts and before-and-after drawings work hand in hand to provide the student with a clear conceptual picture of what's going on *before* leaping to a calculation. If a student can articulate which kinds of energies exist before an interaction and how these energies change because of the interaction (increasing or decreasing), they are much less likely to make errors in the numerical solution.

These initial simple examples immediately raise questions. Is mechanical energy always conserved? If not, when is it and when is it not? It's important to let students know that the energy story has many pieces and that they can't learn everything at once, so you're starting with the simplest situation and will gradually add new pieces of information. This simplest situation, where mechanical energy is conserved, is called an "isolated system," and it consists of two or more objects interacting with each other (these form the system) but not interacting with the rest of the universe (often called "the environment").

This idea can be represented pictorially with the *interaction diagram* of Figure 3. We see that energy within the system can be transformed from one type to another—kinetic to potential or vice versa—but (1) there are no external interactions (no energy is exchanged with the environment); and thus (2) the total energy in the box doesn't change. The drawing for an isolated system is so simple that it hardly seems worthwhile, but we'll soon expand upon this visual representation of interactions to show the "basic energy model" of mechanics and the "thermodynamic energy model."

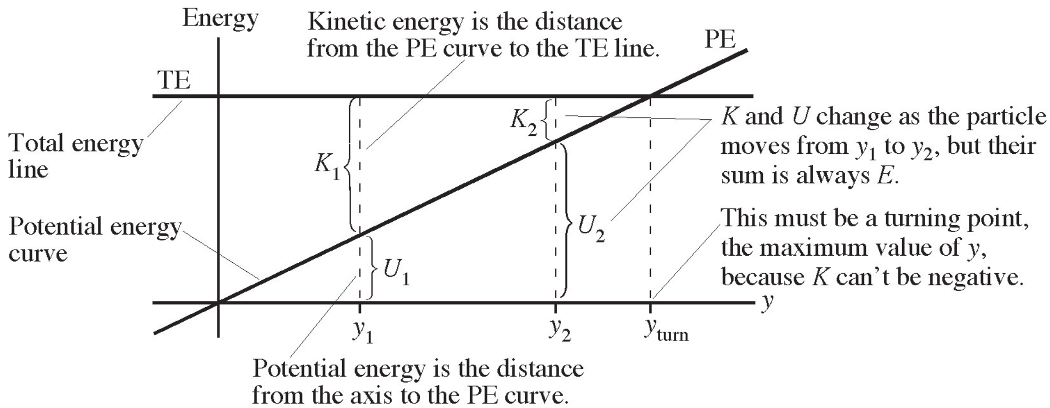
FIGURE 3: AN INTERACTION DIAGRAM FOR AN ISOLATED SYSTEM



Note: For gravity problems such as the one involving Christine on her sled, the objects making up the system are Christine (with her sled) plus the earth as a whole. Christine alone is *not* an isolated system because gravity would then be an external force. Recognizing that gravitational potential energy is an energy of the earth-plus-object system is a subtle but important point. Asking students "What's the system?" and "Is this an isolated system?" can help make the point.

Because potential energy is a function of position, another important representation of energy in an isolated system is the *energy graph*. An energy graph shows two things: the *potential energy curve* $U(x)$ or $U(y)$, which varies with position x or y, and the *total energy line* $E = K + U$, which is horizontal to represent constant total energy. Figure 4 shows and interprets the energy graph for a particle moving under the influence of gravity, where $U(y) = mgy$ is graphed as a straight line with positive slope. This could be another representation of Christine on her sled, or it could equally well be a particle in free fall. Because K cannot be negative, the particle cannot be at a position where the PE curve is above the TE line. Their crossing point represents a *turning point* where instantaneously K (and thus v) is zero.

FIGURE 4: AN ENERGY GRAPH FOR A PARTICLE UNDER THE INFLUENCE OF GRAVITY

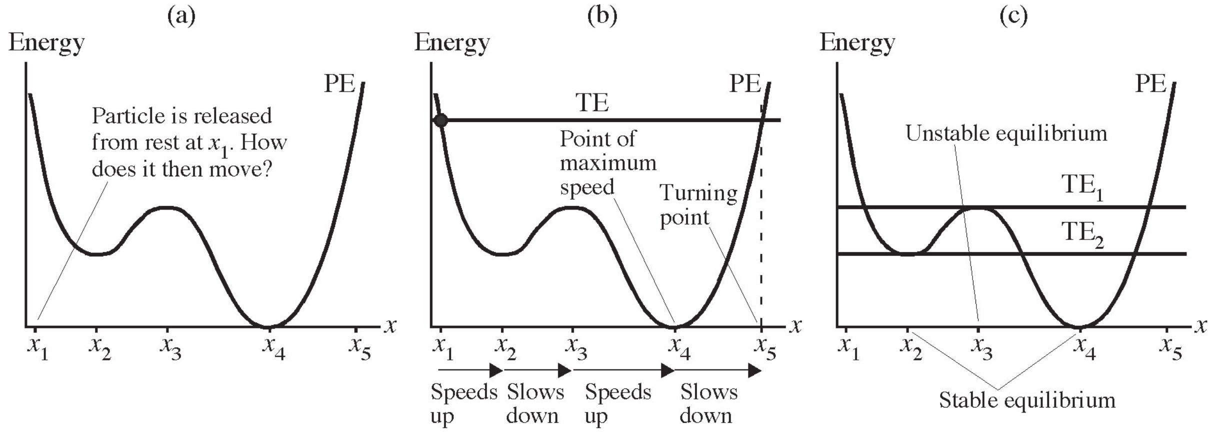


If a particle of mass m is shot straight up from $y = 0$, it slows down (decreasing K) as y increases because kinetic energy is being transformed into gravitational potential energy. It reaches a maximum height at the turning point where the PE curve crosses the TE line ($K = 0$), then reverses direction. We can see that it speeds up as y decreases because K is increasing while U decreases.

Note: The TE line is under your control. You can move the TE line up by shooting the particle up with a larger initial speed. Conversely, a smaller initial speed is represented by a lower TE line and thus a turning point at a smaller value of y.

Now imagine drawing the potential energy curve of Figure 5a on the board, telling students that a particle is released from rest at x_1, and asking them to *describe* its subsequent motion. That is, and where is it speeding up, where is it slowing down, where does it have maximum speed, and where (if anywhere) does it reverse direction? Students (and many teachers!) find this *extremely* challenging. The difficulty is partly a matter of interpreting information presented graphically and partly a matter of not understanding the essential idea of energy conservation—that potential and kinetic energy can be transformed back and forth as long as the total energy doesn't change. The good news is that spending a class period practicing a few examples like this produces remarkable gains in conceptual understanding of energy.

FIGURE 5: INTERPRETING ENERGY GRAPHS

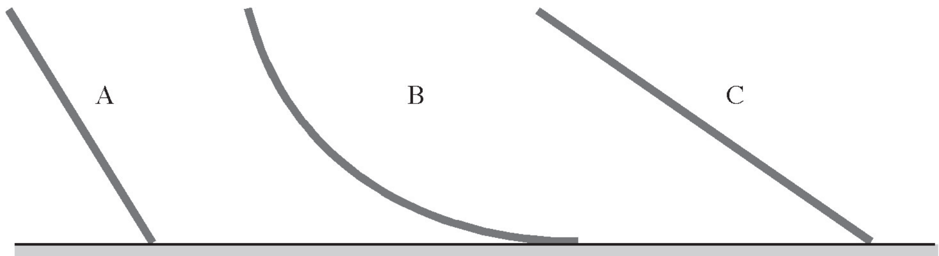


In this case, the particle is released from rest ($K = 0$) at x_1, so the energy at this point is purely potential and thus the TE line crosses the PE curve at this point. This is shown in Figure 5b. Upon release, the particle can't move to the left, because that would make K negative, so it begins moving to the right, speeding up as U decreases and K increases. After passing x_2, U begins increasing and so the particle slows down as kinetic energy is transformed back to potential energy. It reaches a minimum of speed—but not zero!—at x_3, then speeds up until reaching maximum speed at x_4 (K is maximum where U is minimum), then slows until reaching a turning point—instantaneously at rest—at x_5. The particle then reverses direction and goes through the same process of speeding up and slowing down until it returns to x_1. But it won't stop there. The particle will continue to oscillate back and forth between x_1 and x_5 like a marble in an oddly shaped bowl.

A final use of energy graphs is for understanding stable and unstable equilibria, as shown in Figure 5c. A particle is at rest ($K = 0$) if the total energy line touches the PE curve. A minimum in the potential energy (points x_2 and x_4) is a stable equilibrium point because a small disturbance—a very small increase in the total energy—would merely cause a very small oscillation around the equilibrium point. Thus, a particle with total energy TE_2 has a stable equilibrium point at x_2. A potential energy maximum is also a point of equilibrium, but it's like a pencil balanced on its point: It is static if the balance is 100 percent perfect, but even an infinitesimal disturbance will cause the particle to head off to distant values of x. x_3 is a point of unstable equilibrium for a particle with total energy TE_1.

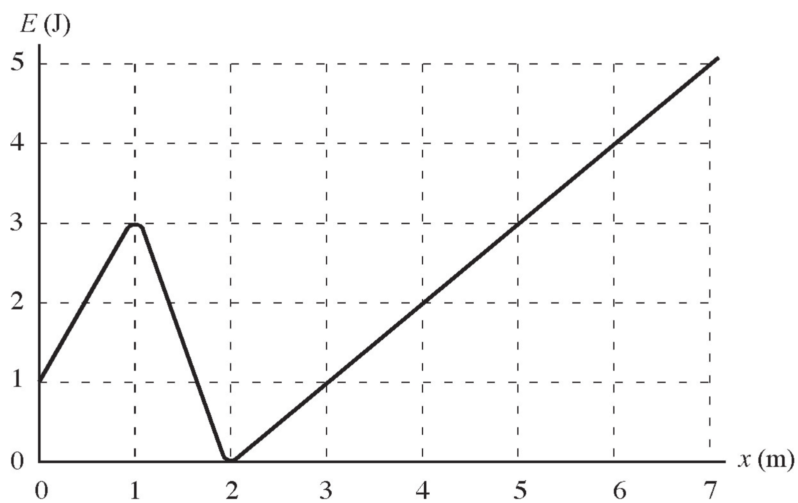
Energy graphs reappear in AP Physics C: Electricity and Magnetism (E&M), where they often become graphs of electric potential rather than potential energy, and in quantum physics, where we talk about "a particle in a potential well." I've found that most students in a modern physics class have no idea what the typical textbook pictures of a potential well represent because they were never asked to interpret graphs like these in mechanics. Helping students learn to reason with energy graphs will help their understanding of energy transformations in mechanics *and* help them be better prepared for E&M and quantum physics.

Assessment Question 1.1: A child slides down the three frictionless slides A, B, and C. Each has the same height. Rank in order, from largest to smallest, the child's speeds v_A, v_B, and v_C at the bottom. Explain your reasoning.



Answer: $v_A = v_B = v_C$. Mechanical energy is conserved. The increase in kinetic energy depends only on the decease in height, not the shape of the path along which the decrease occurs.

Assessment Question 1.2: A particle with the potential energy shown in the graph is moving to the right. It has 1.0 J of kinetic energy at x = 1 m. Where is the particle's turning point? Explain.



Answer: x = 6 m. The potential energy at x = 1 m is U = 3.0 J. Thus the total energy is $E = K + U$ = 4.0 J. Draw a horizontal TE line at 4.0 J. The TE line crosses the PE curve at x = 6 m.

Lesson 2: Work and Thermal Energy

Objective: Work is the mechanical transfer of energy between a system and the environment. The work done by dissipative forces increases the system's thermal energy.

Representations: Before-and-after pictorial representations, bar charts, interaction diagrams, atomic representations.

The definition of work—whether it's a simple "force times distance" or the integral of a variable force—does little to convey the significance of work. What's the point? Lesson 1 focused on isolated systems, but most systems are not isolated. Work is the mechanical transfer of energy to or from a system as it interacts with its environment—that is, energy transferred by pushes and pulls. Thus, "work" needs to be seen as one of two important ways (the second, introduced later, is "heat") by which a system can change its energy. That's the point!

The basic definition of work is for a force acting on a single particle, a restriction that's easily forgotten. When a force acts on a particle, the particle's only option is to change velocity and (except for centripetal forces) to change kinetic energy. This is what the work–energy theorem $\Delta K = W$ tells us. In words, "work" is the energy transferred to a one-particle system by forces, and this energy is used entirely to change the particle's kinetic energy.

Details of how to calculate work in different circumstances are very important, and students are well known to have difficulties when work is zero or negative. Most of those issues are beyond the scope of this essay, but two points are worth emphasizing to students. First, no work is done unless a particle undergoes a displacement. No amount of pushing or pulling does work on a particle that remains at rest. Second, work is positive (energy moves from the environment to the particle as the particle speeds up) when the force (or a component of the force) is in the direction of the particle's displacement; work is negative (energy moves from the particle to the environment as the particle slows down) when the force is opposite the particle's displacement. The key idea from the big-picture perspective is that work—however it's calculated—is how we use mechanical means, pushes and pulls, to change a particle's energy.

All textbooks describe the work done by conservative and nonconservative forces. (Again, details must be found elsewhere.) This leads to the common assertion that the law of conservation of energy is

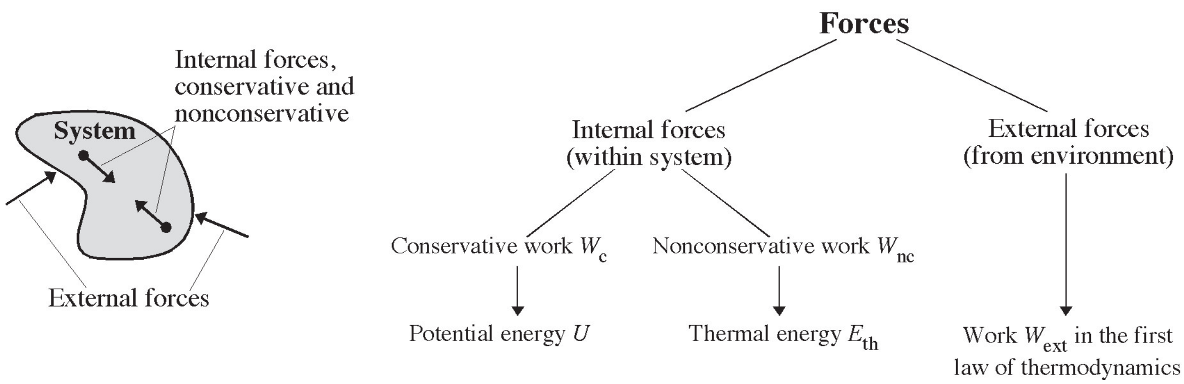
$$\Delta K + \Delta U + \Delta E_{\text{th}} = 0$$

where E_{th} is the system's thermal energy. This statement, as usually presented, is not true. It has not properly distinguished *transfer* of energy from *transformation* of energy.

Simple examples are easily found. If you push a book across a level table at constant speed, $\Delta K = 0,$ $\Delta U = 0,$ and $\Delta E_{\text{th}} > 0.$ Their sum is not zero. If you pick up the book and place it on a high shelf, $\Delta K = 0,$ $\Delta U > 0,$ and $\Delta E_{\text{th}} = 0.$ Their sum is not zero.

Understanding what's going on, and why the standard statement of energy conservation isn't true, provides an important link between energy in mechanics and energy in thermodynamics. Consider the situation in Figure 6. Here we've identified a system of two or more interacting particles. The internal forces within the system may be either conservative forces (gravity, springs, electric forces) or dissipative nonconservative forces (friction). Equally important, the environment may exert *external forces* on the particles in the system.

FIGURE 6: THE MANY FACES OF WORK



As noted above, work is the energy transferred to single particles by forces. The work–energy theorem summed over all the particles in the system is

$$\Delta K = W_{\text{total}} = W_{\text{c}} + W_{\text{nc}} + W_{\text{ext}}$$

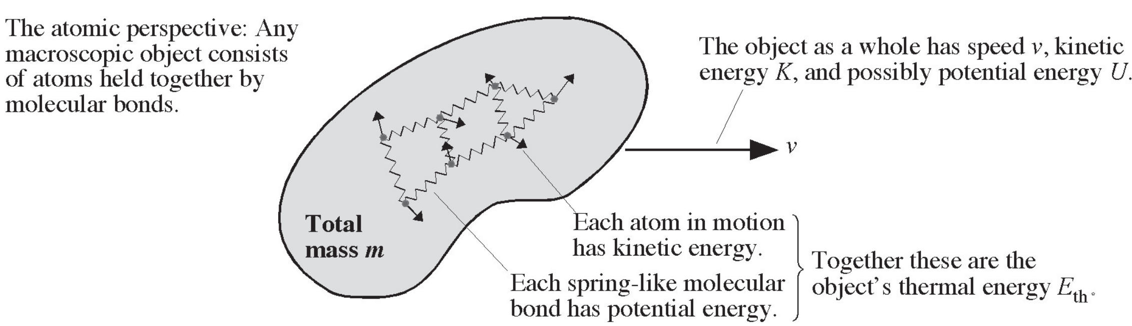
where *K* is the total kinetic energy of all the particles and $W_{\text{c}},$ $W_{\text{nc}},$ and W_{ext} are the work done by internal conservative forces, nonconservative forces, and external forces, respectively. So far there's no potential energy and no thermal energy, just particles whose kinetic energies are changing as forces do work on them.

Potential energy appears because the work done by conservative forces can be associated with a potential energy via $\Delta U(i \to f) = -W_{\text{c}}(i \to f),$ where the notation means the work done or the change in potential energy as the system particles move from initial positions *i* to final positions *f*. Refer to any introductory textbook as to why the minus sign is included in the definition. In essence, potential energy is "precomputed work," which we can do since we don't need to know the particles' trajectories to calculate the work done by a conservative force. If you use a potential energy associated with a conservative force, make sure that you don't also compute the work done by that force because you would then be double-counting!

Similarly, the work done by friction or other dissipative, nonconservative forces *within the system* is associated with an increase in the system's thermal energy: $\Delta E_{\mathrm{th}} = -W_{\mathrm{nc}}$. When a box slides to a halt after being pushed across the floor, students will tell you that the initial kinetic energy "went to heat." That's a correct understanding of the basic idea, but an incorrect use of terms. Heat—to be defined upon reaching thermodynamics—is a transfer of energy, not "hotness." Friction makes things hotter, but we want to associate the ideas of "hotter" and "colder" (higher and lower temperatures) with an increase or decrease of "thermal energy." Note that the work done by friction is always negative because the force direction is opposite the displacement, so the minus sign in the definition makes ΔE_{th} positive: Friction always causes an increase in thermal energy.

Explaining thermal energy is often the first opportunity to use an *atomic representation* like the one in Figure 7. Atomic representations will be important later in thermal physics, E&M, and, of course, modern physics when we want to explain that many *macroscopic* properties of objects can be understood in terms of the *microscopic* behavior of vast numbers of atoms. In this case, we want students to understand that thermal energy is kinetic and potential energy, but now the kinetic energy of moving atoms and the potential energy of stretched/compressed spring-like molecular bonds. This is a very real energy, but distinct from the macroscopic center-of-mass energy of the object as a whole. Increasing an object's temperature makes the atoms move faster and the molecular bonds vibrate with larger amplitudes, so "hotter" really means "more thermal energy."

FIGURE 7: THE ATOMIC REPRESENTATION OF THERMAL ENERGY



In using thermal energy, *both* objects experiencing friction must be part of the system. For a box sliding across a rough surface, both the box and the surface are part of the system. This way, the friction forces are internal forces within the system, such as those shown in Figure 6. Because both the box *and* the surface get hot, ΔE_{th} is the *combined* thermal energy increase of both. We can't determine the separate thermal energy increases—at least not without a lot more information (masses, specific heats, etc.) and a much more detailed analysis.

Using $W_{\mathrm{c}} = -\Delta U$ and $W_{\mathrm{nc}} = -\Delta E_{\mathrm{th}}$ in the work–energy theorem, we have

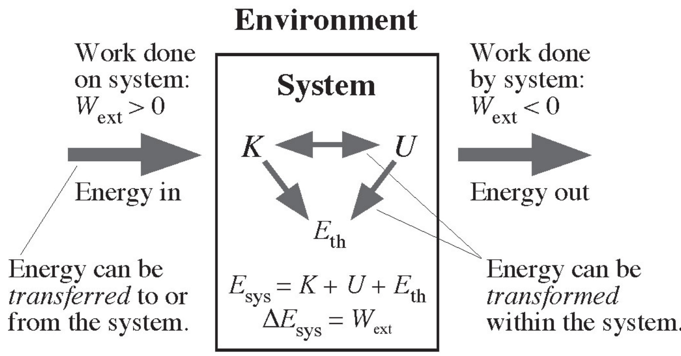
$$\Delta K + \Delta U + \Delta E_{\mathrm{th}} = W_{\mathrm{ext}} .$$

If all the forces are internal forces—an isolated system—then the simple energy conservation equation $\Delta K + \Delta U + \Delta E_{\mathrm{th}} = 0$ is valid. In an isolated system we have only *transformations* of energy: potential to kinetic (a falling rock), kinetic to thermal (a sliding box), etc.

In fact, we can say that the mechanical energy $E_{\mathrm{mech}} = K + U$ is conserved for a system that is both isolated ($W_{\mathrm{ext}} = 0$) and nondissipative ($W_{\mathrm{nc}} = 0$), whereas it's the *total energy* $E_{\mathrm{sys}} = K + U + E_{\mathrm{th}}$ that is conserved for a system that is merely isolated but might have friction. This is an important conclusion. An excellent class exercise is to toss out a variety of situations—a boy going down a frictionless slide, a boy going down a rough slide, a boy being pulled down a rough slide by a rope—and asking what, if anything, is conserved. Correctly identifying conserved quantities is essential to formulating a correct problem-solving approach.

If external forces act on the system, we have the situation shown in the revised interaction diagram of Figure 8. Compare this to the simpler interaction diagram of an isolated system in Figure 3. You can see that our ideas about energy—what it is and what it does—are expanding.

FIGURE 8: THE BASIC ENERGY MODEL, A REVISED INTERACTION DIAGRAM



I like to call Figure 8 the "basic energy model." It's not everything about energy, because we haven't yet included heat, but this representation of interactions contains everything we need to know for understanding and using energy in mechanics. Further, this model can easily be extended to include heat, as we will do in Lesson 3. In essence, this diagram is a pictorial representation of the full energy story in mechanics. In particular:

- Energy is *transferred* between the system and the environment as the work done by external forces. Negative work—a real possibility—means that the system's energy decreases.
- Energy is *transformed* within the system. Physically, this is due to the work of internal interaction forces, conservative and nonconservative, but it's more useful to think of these as changes in potential energy and changes in thermal energy.
- The full conservation of energy statement in mechanics is

$$\Delta E_{sys} = \Delta K + \Delta U + \Delta E_{th} = W_{ext}.$$

  A student who understands that this is the mathematical representation of Figure 8 is well on the way to successfully understanding and using energy.

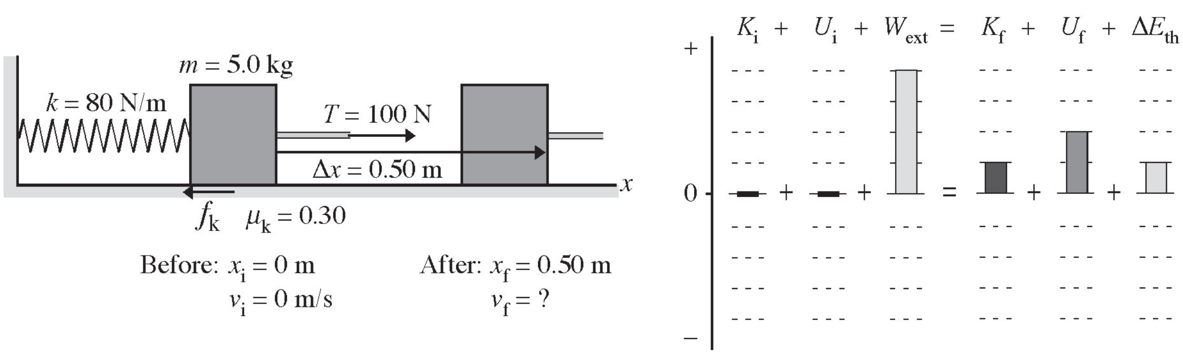
Note: It is important to include thermal energy when discussing mechanical energy. After all, friction is a real part of everyday experience, and understanding what friction does to energy is an important idea. Unfortunately, most textbooks do not discuss thermal energy at this point in the course, or at best only mention it in passing. This leaves the student able to apply energy ideas only in ideal situations of no friction. Further, exclusion of thermal energy misses an important opportunity for developing the "big picture" of what energy is all about.

To see all the pieces in play, consider the following problem that is quite a bit more complex than that of Christine and her sled in Lesson 1:

> A 5.0 kg box is attached to one end of a spring that has a spring constant of 80 N/m. The other end of the spring is attached to a wall. Initially the box is at rest at the spring's equilibrium position. A rope with a constant tension of 100 N pulls the box away from the wall. The coefficient of kinetic friction between the box and the floor is 0.30. What is the speed of the box after it has moved 50 cm?

Figure 9 shows both a before-and-after pictorial representation and an extended energy bar chart. This is not a problem likely to be solved by plug-and-chug into an equation, but it is not a terribly hard problem *if* you use these visual representations to understand what it going on. We've chosen the system to be box + spring + floor; the spring has to be inside the system to use the elastic potential energy of the spring, and the floor has to be inside the system because friction is going to increase the thermal energy of both the box *and* the floor. The rope's tension is then an external force doing work on the system, transferring energy into the system. That outside energy increases the box's kinetic energy, increases the spring's potential energy, and increases the thermal energy of the box and floor.

FIGURE 9: A BEFORE-AND-AFTER PICTORIAL REPRESENTATION AND AN ENERGY BAR CHART



The full energy conservation statement is $\Delta E_{sys} = \Delta K + \Delta U + \Delta E_{th} = W_{ext}$. The rope does external work $W_{ext} = T\Delta x$, the spring's potential energy increases by $\Delta U = \frac{1}{2}k(\Delta x)^2$, and the thermal energy increases by $\Delta E_{th} = -W_{fric} = -(-f_k\Delta x) = \mu_k mg\Delta x$. These can be used to compute the box's increase in kinetic energy and thus to find $v_f = 3.6$ m/s. A student who successfully solves a problem like this has come a long way toward a full understanding of energy in mechanical systems.

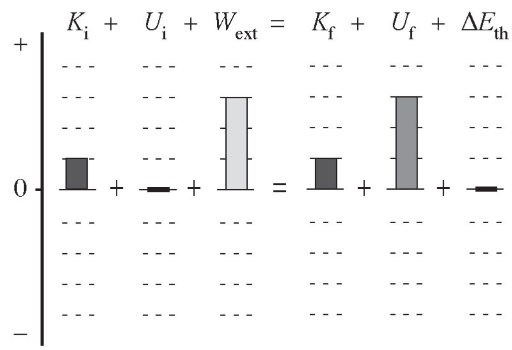
Assessment Question 2.1: A child slides down a playground slide at constant speed. The energy transformation is

a. $U \rightarrow K$ b. $K \rightarrow U$ c. $U \rightarrow E_{th}$ d. $K \rightarrow E_{th}$
e. There is no transformation because energy is conserved.

Answer: c. $U \rightarrow E_{th}$. Constant speed means that kinetic energy isn't changing (at least not after a brief increase at the start). Potential energy is decreasing while the work done by friction is increasing the thermal energy of the slide and the child's pants.

Assessment Question 2.2: A rope lifts a box straight up at constant speed. Show the energy transfers and transformations on an energy bar chart.

Answer:



The tension in the rope is an external force doing work on the box, increasing its potential energy. The box has kinetic energy—it's moving—but the kinetic energy is not changing.

Lesson 3: Work, Heat, and the First Law of Thermodynamics

Objective: The first law of thermodynamics is a continuation of the study of energy in mechanics. It describes how a system interacts both mechanically and thermally with its environment.

Representations: Interaction diagrams, *pV* diagrams.

It is hoped that the study of energy in mechanics ended with the energy conservation statement $\Delta E_{sys} = \Delta K + \Delta U + \Delta E_{th} = W_{ext}$. Upon starting thermodynamics, it's easy to point out that this equation can't be the full story. Consider putting a pan of water on the stove. No external work is done and neither the kinetic nor potential energy of the water as a whole changes, yet the water gets hotter—its thermal energy increases. If you earlier introduced the basic energy model of Figure 8 and talked about the distinction between energy transformations (within the system) and energy transfers (to or from the environment), it's clear that the water is getting hotter ($\Delta E_{\text{th}} > 0$) due to some kind of energy transfer from the environment.

The energy conservation statement from mechanics is not wrong; it is merely incomplete. Work, a *mechanical process* using pushes and pulls, is not the only way energy can be transferred to or from the system. We can now define "heat" as a *thermal process* in which energy is transferred from a hotter object to a colder object due to a temperature difference. Mechanical and thermal processes are the two broad categories for how a system exchanges energy with its environment.

If heat is included as a second way to transfer energy, the energy conservation statement becomes

$$\Delta E_{\text{sys}} = \Delta K + \Delta U + \Delta E_{\text{th}} = W_{\text{ext}} + Q\,.$$

This equation has a name. It is the first law of thermodynamics, but it is not yet expressed as it usually is within thermodynamics. Nonetheless, it's important to start your discussion of thermodynamics this way because it shows that *thermodynamics is simply a continuation of mechanics*. We've introduced a new process for transferring energy, but the framework for thinking about energy—in which we invested a lot of time back in mechanics—is still valid.

Surprisingly, few textbooks make this connection. Energy in thermodynamics is introduced as if it has little or no connection to energy in mechanics, so it comes as no surprise that almost no student sees the connection.

To finally reach the usual statement of the first law, we can note that the systems of interest in mechanics are *stationary* containers of gas or liquid whose center-of-mass mechanical energy does not change; that is, $\Delta K + \Delta U = 0.$ It is not that we couldn't use the heat of burning fuel to launch the system as a whole into the air, but that's not the focus of standard thermodynamics. If we restrict ourselves to stationary systems, then $\Delta E_{\text{sys}} = \Delta E_{\text{th}};$ the only part of the system's energy that can change is its thermal energy.

Further, the work done in thermodynamics is *always* the external work of piston rods or other forces that expand or compress gases. We don't need to distinguish between the work of external forces and the work of internal forces, so the subscript "ext" on W_{ext} is superfluous and can be dropped. With these two changes, the above energy conservation statement is reduced to

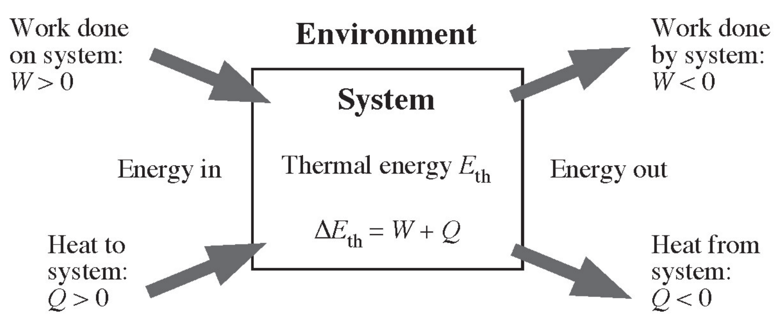
$$\Delta E_{\text{th}} = W + Q \text{ (first law of thermodynamics).}$$

This is, indeed, how the first law of thermodynamics is stated in my textbooks and a few other textbooks. Unfortunately, you'll often see the first law written as $Q = \Delta U + W$. Why is this unfortunate?

- Many textbooks use U to represent thermal energy, even though just a few weeks earlier we used U to represent potential energy. And we'll do so again when we get to electric potential energy. Using U for thermal energy is guaranteed to confuse students.
- With no explanation, many textbooks switch from having W represent work done *on* the system, as it was in mechanics, to work done *by* the system. The magnitudes are the same, but this flips the sign of W and thus moves it to the other side of the equation. The pedagogical advantage of writing $\Delta E_{\text{th}} = W + Q$ is that it places work and heat on an equal footing: They're both processes by which energy is transferred between the system and the environment, and a positive quantity indicates that energy is flowing into the system. More and more scientific organizations are recommending that work in thermodynamics be defined as it is in mechanics—as work on the system—but textbooks have been slow to change.
- With a change in notation and a change in the definition of work, the link is broken between energy in thermodynamics and energy in mechanics. A great opportunity to understand the big picture, what energy is all about, has been lost unless you intervene.

Figure 10, another interaction diagram, is what I call the "thermodynamic energy model." Compare this to Figure 8 to see how the energy ideas from mechanics all carry over to thermodynamics.

FIGURE 10: THE THERMODYNAMIC ENERGY MODEL



Note: We often refer to "work done on the system" when a gas is compressed and to "work done by the system" when a gas expands. Although this terminology is useful, it is also misleading. Research has found that many students think *either* the environment does work *or* the system does work, *but not both*. Because of Newton's third law, $W_{\text{system}} = -W_{\text{environ}}$, so the environment does work *and* the system does work, but with opposite signs. Work "on the system" refers to a situation where $W_{\text{environ}} > 0$ (and thus $W_{\text{system}} < 0$), whereas work "by the system" means $W_{\text{system}} > 0$ (and thus $W_{\text{environ}} < 0$). The work in the first law, as I've written it, is W_{environ}. When we informally say "work is done by the system," we're referring to a situation where the work done on the system—the quantity you need to use in calculations—is negative. This is a subtle point, but a very important point to discuss with students.

As in mechanics, it's very worthwhile to pose situations and ask whether *W*, *Q*, and ΔE_{th} are positive, negative, or zero. For example:

- You drive a nail into wood with a hammer ($W > 0,\ Q = 0,\ \Delta E_{\text{th}} > 0$). The nail and hammer get hot, but this has nothing to do with heat.
- You turn on a flame under a cylinder of gas, and the gas undergoes a slow, isothermal expansion ($W < 0,\ Q > 0,\ \Delta E_{\text{th}} = 0$). Heat doesn't always mean that something gets hot.
- High-pressure gas in a well-insulated cylinder pushes a piston out very rapidly (an adiabatic process with $W < 0,\ Q = 0,\ \Delta E_{\text{th}} < 0$). Temperature can change without the presence of heat.

Remember that *W* is work *on* the system, so gas expansions have $W < 0$.

Students have far more trouble with questions like this than you might expect. They associate "heat" with "hotness" and thus make many errors when a temperature change is due to work rather than heat. But it's much better to confront these issues early, when heat and the first law are introduced, than to have students making these errors when you later get to heat engines. Using the pictorial representation of Figure 10 is very useful for helping students think about situations carefully rather than falling back on preconceived notations about heat and temperature. Is there a mechanical interaction? Is there a thermal interaction in which the system and environment exchange energy because of a temperature difference? What conclusions can we draw from the first law? These are the questions you want your students to focus on.

Ideal gases introduce a new representation, the *pV* diagram. Many textbooks don't introduce *pV* diagrams until they get to heat engines, but that's too late. Students have a lot of trouble understanding and using *pV* diagrams, and it's better to introduce them when you first introduce the ideal gas law, and then use them as you do standard gas law problems about pressure changes and temperature changes. Then they'll be a familiar and useful tool when you get to heat engines rather than one more new idea in a difficult chapter.

Our focus here is on using *pV* diagrams to illustrate the work *W* in the first law. Because we're using *W* as the work *on* the system, we have

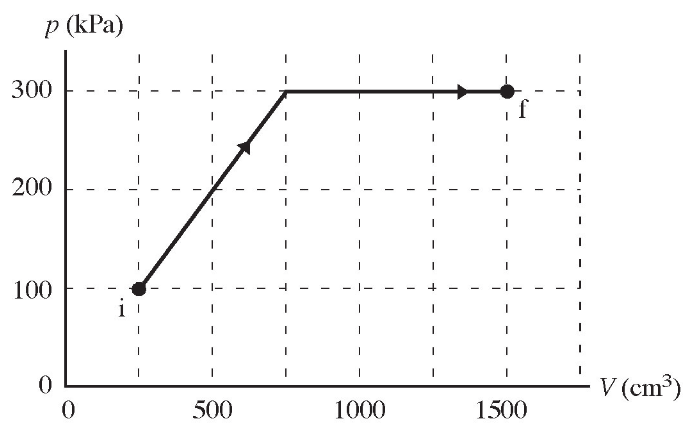
$$W = -\int_{V_i}^{V_f} p\,dV = -(\text{area under the } pV \text{ curve between } V_i \text{ and } V_f).$$

Many textbooks, of course, define *W* as the work *by* the system and thus calculate *W* as the *positive* area under the curve. If your book does so, it's probably best to follow the book rather than confuse students by using a different sign convention in class, even though doing so (as discussed above) breaks the connection with mechanics.

Example:

How much work is done on the gas in the ideal gas process shown in Figure 11?

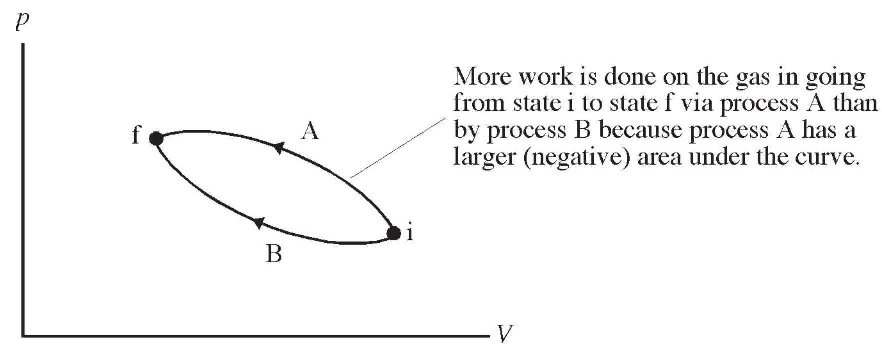
FIGURE 11: AN IDEAL GAS PROCESS



The work done on the gas is the negative of the area under the curve. Here we can find the area geometrically rather than explicitly carrying out the integration. The area from 500 cm^3 to 1,000 cm^3 can be divided into a rectangle of height 100 kPa and a triangle of height 200 kPa. The area under the curve from 1,000 cm^3 to 1,500 cm^3 is a simple rectangle. Volumes must be converted to SI units of m^3, in which case the product of Pa and m^3 is J. (It's worth having your students verify this by multiplying through the units.) Calculation of the three areas gives $W = -325$ J. Work *on* the system is negative during an expansion because the force exerted by the piston rod is opposite the displacement of the piston. Alternatively, the work done *by* the gas is +325 J.

The work done on a gas as it goes from an initial state *i* to a final state *f* depends on the process by which the gas changes state. For example, Figure 12 shows two processes by which gas is compressed from initial state *i* to a final state *f* with smaller volume. Because *W* is the negative of the area under the curve and process A has a larger area (a negative area because we're integrating from right to left in this case), more work is done to compress the gas using process A than in using process B even though the final result is the same.

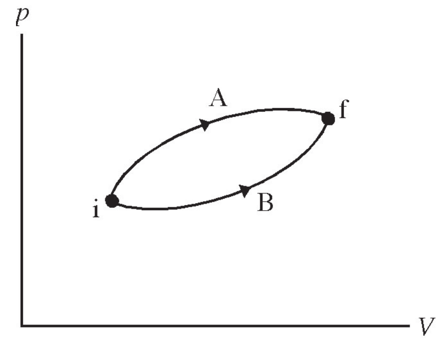
FIGURE 12: WORK DEPENDS ON THE PROCESS



Suppose students are shown this diagram and asked, "Which process does more work on the gas? Or are they equal?" Research has found that a majority of students incorrectly answer "equal." There seem to be three factors that enter into their failure to understand a crucial issue. First, they are confusing a "process quantity," work, with "state variables." It's true that Δp, ΔT, and ΔE_{th} depend only on the end points, not the process, because they depend only on the state of the gas. This reasoning does not apply to *W* and *Q*. Second, they are misapplying a statement—"the work done by a conservative force does not depend on the path"—that they learned in mechanics. That statement refers to a physical trajectory between two points in space, and it is the basis of defining potential energy. The external force in thermodynamics is not a conservative force, and the "path" goes through the *pV* diagram and shows a sequence of states, not a physical path through space. Third, most textbooks don't give enough emphasis to *pV* diagrams, so students are not able to understand a situation by "reading" the *pV* diagram. The ability to interpret a *pV* diagram is strongly correlated with having a good grasp of the principles of thermodynamics, so it is important to help students learn to interpret *pV* diagrams.

As a follow-up question: Which process seen in Figure 13, A or B, requires the larger amount of heat? Or is the heat the same for both processes? Explain.

FIGURE 13: WHICH PROCESS, IF EITHER, REQUIRES MORE HEAT?



A majority of students think that $Q_A = Q_B$. The temperature increase is the same for both processes, so shouldn't they require the same heat? The issue here, as with Figure 12, is that heat (as well as work) depends on the process—on the path through the *pV* diagram. Because temperature and thermal energy *are* state variables, their change depends only upon the end points. That is, $(\Delta E_{th})_A = (\Delta E_{th})_B$. Applying the first law, it then must be true that

$$Q_A + W_A = Q_B + W_B.$$

The work done *on* the gas is negative for an expansion, so $Q_A - |W_A| = Q_B - |W_B|$. Based on the area under the curve, $|W_A| > |W_B|$. Consequently, we conclude that $Q_A > Q_B$.

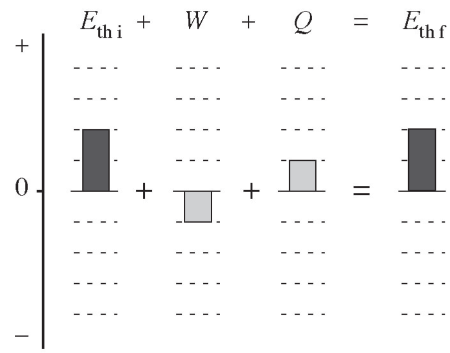
Assessment Question 3.1: A gas cylinder and piston are covered with heavy insulation. The piston is pushed into the cylinder, compressing the gas. In this process, the gas temperature

a. increases b. decreases c. doesn't change
d. There's not enough information to tell.

Answer: a. increases. The first law is $\Delta E_{th} = W + Q$. No heat flows in or out due to the insulation. However, the piston does work *on* the gas, so $W > 0$. Thus $\Delta E_{th} > 0$, and an increased thermal energy implies an increased temperature.

Assessment Question 3.2: What type of gas process is represented by the following bar chart?

a. isochoric b. isobaric c. isothermal d. adiabatic
Is the process e. a compression or f. an expansion?



Answer: c and f; isothermal expansion. The thermal energy does not change, which means that the temperature does not change. *W* is negative, indicating an expansion in which the system does work on the environment: $W_{\text{system}} > 0$ and thus $W = W_{\text{environ}} < 0$.

Lesson 4: Using Energy in Thermodynamics

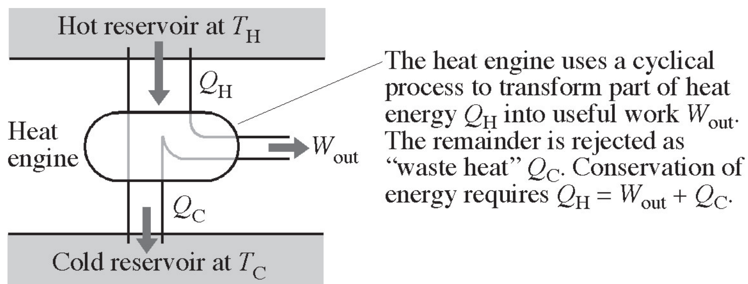
Objective: To understand energy flow in heat engines.
Representations: Energy-flow diagrams, *pV* diagrams.

A heat engine is a very abstract idea. Real engines—such as steam engines or gasoline engines—are complicated gadgets that engineers work with. Our goal in physics is to understand the physical principles behind *any* engine that allow it to do useful work, regardless of the details of operation. Thus a heat engine is a *model* of real engines in much the same way that a particle is used to model the motion of cars or rockets. This is an important point to stress because many students have a hard time grasping the whole concept of a heat engine.

A heat engine is a device that uses a *cyclical process* to transform heat energy into work. Analyzing heat engines is, in many ways, the culmination of the "energy story"—what we've been working toward since first introducing kinetic and potential energy. At the same time, we find that energy concepts alone are insufficient for understanding heat engines. Energy conservation alone does not prevent heat energy flowing from cold to hot, nor does it prohibit a heat engine having 100 percent efficiency. The second law of thermodynamics is the first new law of physics encountered since leaving Newtonian mechanics, and a full understanding of heat engines requires both the first *and* second laws of thermodynamics. Nonetheless, this essay is about energy, so it will focus only on the energy aspects of heat engines.

The *energy flow diagram* of Figure 14 is one more way to represent the first law of thermodynamics. The idea is to think of energy flowing like water though pipes; the heat engine itself neither makes nor destroys energy, so the total energy flowing out of the side and bottom "pipes" must exactly equal the energy entering through the top "pipe."

FIGURE 14: THE ENERGY FLOW DIAGRAM OF A HEAT ENGINE



To see how Figure 14 is a pictorial representation of the first law, start by writing the first law for one complete cycle of the engine:

$$(\Delta E_{th})_{cycle} = W_{cycle} + Q_{cycle} = -W_{out} + (Q_H - Q_C)$$

It will be convenient to work with positive quantities, so define $W_{out} = -W_{cycle}$ as the work done *by* the system in one cycle. Because a heat engine is a practical device for doing work, in this case it does make more sense to use the work done *by* the system, which is positive for a heat engine. Over the course of a cycle, sometimes heat flows *into* the engine from a reservoir at a higher temperature ($Q > 0$) and sometimes heat flows out into a reservoir at a colder temperature ($Q < 0$). The sum of all the positive heats (which may have more than one term due to different parts of the cycle) is called Q_H. The absolute value of the sum of all the negative terms is Q_C; in other words, Q_C is the *magnitude* of the heat flowing out of the engine, and thus is positive. With these definitions, we get the right-hand side of the above equation.

The pivotal step in the logic is that $(\Delta E_{th})_{cycle} = 0$. This is why we *define* a heat engine as using a cyclical process. Because E_{th} is a state variable, it returns to its initial value when the engine has completed a full cycle and returned to its initial state. The thermal energy can and does change during different portions of the cycle—a fact that many students overlook as they incorrectly try to apply $\Delta E_{th} = 0$ to each part of the cycle—but the changes cancel to give no *net* change over the entire cycle.

If we use $(\Delta E_{\text{th}})_{\text{cycle}} = 0,$ we can now write the first law for the heat engine as

$$Q_{\text{H}} = W_{\text{out}} + Q_{\text{C}}\,.$$

This is exactly what Figure 14 is showing: Heat energy Q_{H} flows in from the hot reservoir. Some of that energy is transformed into useful work, and the remainder is rejected to the cold reservoir. What comes in has to go out, so $Q_{\text{H}} = W_{\text{out}} + Q_{\text{C}}$.

Many students fail to understand this important reasoning, so they really don't know what pictures like Figure 14 are showing. Consequently, they then tend to make lots of sign errors because they don't know which terms are defined to be positive numbers. It's definitely worth going through the reasoning carefully, then—before starting "real" heat engine problems—engaging students in simple exercises such as "How much work is done per cycle by a heat engine that takes in 200 J of heat per cycle and rejects 100 J of heat per cycle?" or "If a heat engine rejects 200 J of waste heat per cycle, how much heat energy from the hot reservoir is required per cycle to do 300 J of useful work per cycle?" You want students to understand that a big part of heat engine analysis is simple bookkeeping—just as energy conservation has been all along. Note the emphasis here on "per cycle."

You can introduce the idea of efficiency at the same time. I define thermal efficiency as

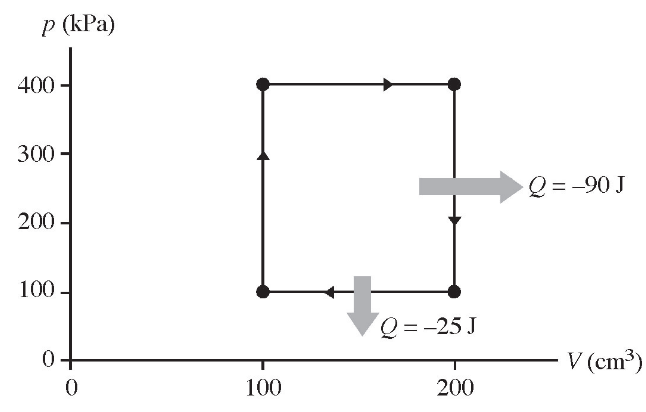
$$\text{thermal efficiency} = \eta = \frac{\text{what you get}}{\text{what you had to pay to get it}} = \frac{W_{\text{out}}}{Q_{\text{H}}}\,.$$

Students especially like this definition; it "makes sense" to them. The fact that the second law of thermodynamics sets an upper limit on η will arise later, but it's best not to get into that at the early stages of discussing heat engines.

Energy flow diagrams and *pV* diagrams are nicely combined in exercises like the following:

What are W_{out}, Q_{H}, and η for the heat engine shown in Figure 15?

FIGURE 15: WHAT IS THE EFFICIENCY OF THIS HEAT ENGINE?

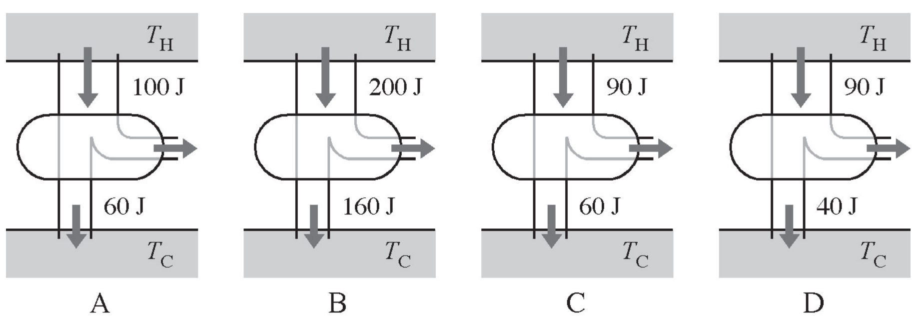


Questions such as this focus on the fundamental ideas of heat engines without getting bogged down in detailed calculations of heat and work. In this case, the two sides of the cycle shown with negative heat are processes where the engine temperature is falling. Thus, $Q_C = 90\text{ J} + 25\text{ J} = 115\text{ J}$. The other two sides are processes where the engine temperature is rising, so those are processes where heat is flowing in from the hot reservoir.

The work done per cycle *by* the engine is the area inside the curve. Because the curve is a simple rectangle, $W_{out} = (300{,}000\text{ Pa}) \times (100 \times 10^{-6}\text{ m}^3) = 30\text{ J}$. We can then use energy conservation to calculate $Q_H = W_{out} + Q_C = 145\text{ J}$ per cycle. Now that we know all the pieces, we find the thermal efficiency to be $\eta = W_{out}/Q_H = 0.21 = 21\%$.

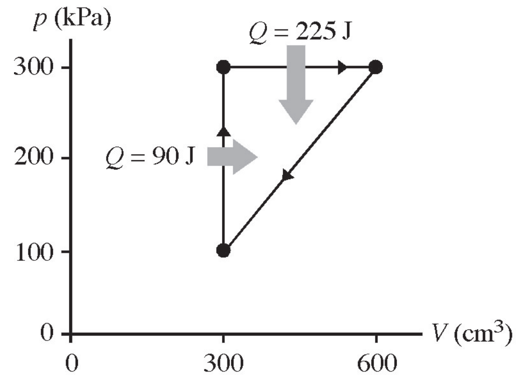
Note: I recommend an early introduction to *pV* diagrams. With sufficient earlier practice, students can now use *pV* diagrams as a useful representation for understanding heat engines. If *pV* diagrams are only now being introduced, students will be trying to use a new and unfamiliar tool for understanding the new and unfamiliar concept of the heat engine. Multiple representations are powerful tools, but they have to be phased in gradually, as the opportunity arises, rather than unloaded on the students all at once.

Assessment Question 4.1: Rank in order, from largest to smallest, the work performed by each of these heat engines. Then rank in order, from largest to smallest, the thermal efficiencies of the engines.



Answer: $W_D > W_A = W_B > W_C$ and $\eta_D > \eta_A > \eta_C > \eta_B$. The work done is $W_{out} = Q_H - Q_C$. This is 40 J, 40 J, 30 J, and 50 J for engines A–D. The efficiency is $\eta = W_{out}/Q_H$. This is 40 percent, 20 percent, 33 percent, and 56 percent for engines A–D.

Assessment Question 4.2: How much heat is exhausted to the cold reservoir by the heat engine represented by this *pV* diagram?



Answer: 285 J. From energy conservation, $Q_C = Q_H - W_{out}$. The two heats shown represent heat flowing in from the hot reservoir, so $Q_H = 315$ J. Work is the area inside the curve, which we can compute as the area of the triangle:

$W_{out} = \frac{1}{2} \times (200{,}000 \text{ Pa}) \times (300 \times 10^{-6} \text{ m}^3) = 30$ J. Thus $Q_C = 285$ J.

Conclusion

This essay has looked at some of the many ways in which energy and energy-related processes can be represented. The emphasis has not been on the mathematical details of working energy problems but rather on the "big picture" of how we think about energy, how we understand the major energy-related concepts, and what we want our students to remember about energy.

Multiple representations of energy knowledge allow students to reach a deeper understanding of what energy is all about. Students have little trouble applying the necessary mathematics *if* they are able to reason their way through a problem. But the converse is not true; there's no evidence that a detailed focus on equations and plug-and-chug problems leads to a deeper understanding of energy.

The different representations of energy are not all that a student must learn, of course, but a student who can easily move between different representations of knowledge is well on his or her way to mastery of the subject.

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