CollegeBoard
Advanced Placement
Program

AP® Physics

2006–2007
Professional Development
Workshop Materials

Special Focus:
Graphical Analysis

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Important Note: The following set of materials is organized around a particular theme, or "special focus," that reflects important topics in the AP Physics course. The materials are intended to provide teachers with resources and classroom ideas relating to these topics. The special focus, as well as the specific content of the materials, cannot and should not be taken as an indication that a particular topic will appear on the AP Exam.

A Note from the Editor

Dolores Gende
Parish Episcopal School
Dallas

The following new set of AP Physics theme materials centers on graphical analysis. The theme has been addressed in six articles created by AP Physics teachers to help their colleagues better prepare their students in the area of graphical analysis. These pieces include instructional strategies and a variety of exercises across many topics in the AP Physics B and C curricula.

In “Graphical Analysis for Physics: An Introduction,” Laurence S. Cain, chair of the Development Committee, underscores the importance of graphical analysis as a skill and a tool in various areas of the AP Physics curriculum. My article, “Graphical Analysis of Motion: Kinematics,” offers an instructional approach to the qualitative and quantitative study of motion in one dimension. The exercises contained in this piece emphasize conceptual understanding of the motion of objects moving at constant speed and objects in accelerated motion. Hasan Fakhruddin’s “Energy Diagrams in Mechanical Systems and the Graphs for Oscillatory Systems” presents a variety of exercises that involve the analysis of energy diagrams and graphs for situations that involve mechanical and oscillatory systems.

Next, in “The First Law of Thermodynamics and P-V Diagrams,” James Mooney discusses thermodynamics, an area that corresponds exclusively to the AP Physics B curriculum. The article includes a thorough discussion of the first law of thermodynamics and the different thermodynamics processes that can be represented and analyzed through P-V diagrams. In “Field and Potential Graphs,” Boris Korsunsky presents an effective didactic approach with examples on the topic of electric force, electric field, and electric potential through the analysis of graphical representations of electric field and graphs of force versus charge. Finally, my “Graphing Analysis in Modern Physics” explores atomic energy levels and the photoelectric effect, topics that belong to the area of atomic and nuclear physics in the AP Physics B curriculum. The paper includes various exercises on energy-level diagrams and the representation and analysis of experimental data of the photoelectric effect.

It is the contributors’ hope that you will find these pieces helpful in covering this topic in the AP Physics classroom.

Graphical Analysis for Physics: Introduction

Laurence S. Cain
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As chair of the AP Physics Development Committee, I am pleased to present these theme materials on graphical analysis. The ability to analyze graphs is an important and necessary skill for AP Physics students. The AP Physics Course Description lists several key abilities evaluated by the AP Exam, including drawing and interpreting graphs and representing data or physical relationships in graphical form. The laboratory section of the exam also requires graphing skills with questions that ask students to "analyze data, including displaying data in graphical or tabular form, fitting lines and curves to data points in graphs, performing calculations with data, or making extrapolations and interpolations from data."[1]

With the publication of these theme materials, the Development Committee is committed to addressing the need for students to have conceptual understanding of this required material.

Students need to be able to think about the material in their physics courses in terms of conceptual, verbal, graphical, and mathematical ideas. As part of these comprehensive skills for understanding the physical world around them, students must be able to perform graphical analysis in its many forms. Thus the AP Physics Exams continue to address the analysis of graphs in all types of questions, including laboratory-related questions. With the use of graphing calculators, students appear to be losing the ability to draw, interpret, and understand graphs. "The calculator does it" has become a constant refrain, but student performance on recent AP Exams leads the Committee to believe that many students don't have the basic physics knowledge to understand what the calculator is doing and why.

In many areas of physics, there also appears to be a disconnect between what students learn in their mathematics courses and how they apply that knowledge in their physics courses. For example, even if students have learned graphing in previous math courses and understand the concept of slope, they may have difficulty understanding that the

[1] 2006, 2007 *AP Physics B, Physics C: Mechanics, and Physics C: Electricity and Magnetism Course Description*, pp. 19 and 20.

slope of a displacement-versus-time graph is the velocity. The AP Physics courses should provide an opportunity to bridge the gap between physics and math for these students.

Problem Areas in Graphical Analysis

There are various broad categories under the general area of graphical analysis. One of these areas involves the straightforward plotting of data. With the advent of graphing calculators, this ability seems to have been deemphasized. Many students have trouble with data plotting, seemingly because they do not understand the fundamentals of graphing and what a graph means. They have difficulty choosing the variables to plot, indicating on the graph what they have plotted, and labeling the correct units. They have difficulty making scales uniform and drawing graphs that may not include the zero on one or both axes if these zeros are not part of the data set.

A second area where students struggle is linearizing data. Students appear to have trouble deciding how to plot a relationship so that a best fit to the data can give information from the slope and the intercept. Many students connect the dots; many choose two data points that are not on the best-fit line or use one point and an inappropriate zero to find a slope; many draw a straight line through data uncritically, even when such a fit is not appropriate; and many choose two points very close together and ignore the full data set when finding a slope. The ability to linearize data requires a good understanding of functions. This is an ability that many students have not developed.

A third area involves the ability to view and interpret graphs that are already given or to predict what a graph will look like. This area spans all topics in physics and requires a good conceptual and mathematical understanding of the underlying physics. Students should be able to interpret graphs and make predictions. With the help of their graphing calculators, they can quickly check their ideas and practice understanding in this area. Particularly important is the ability to interpret position, velocity, and acceleration graphs. The conceptual understanding involved in using slopes and areas to find kinematical variables and the relationships among them is an important ability for students to develop. This understanding sets the stage for the use of graphical analysis later in the AP courses.

Examples of the Problem Areas Observed on Previous AP Exams

A number of examples of student troubles with graphical analysis can be found on the 2005 AP Physics Exams. The Chief Reader's Student Performance Q&A for the AP Physics B Exam[2] points out several problem areas:

- "The areas in which students need work are experimental technique in general **and graphical analysis in particular**" (p. 6, bold added for emphasis).
- 2005 B1 involved the sketching of a graph of velocity versus time given a graph of position versus time. The Q&A states that "the majority of students could draw some kind of graph, but many had problems properly sketching the transitions" (p. 1).
- 2005 B4 was a laboratory question. As part of this question, the students were asked to sketch a graph of intensity versus position for a double-slit interference pattern. From the Q&A: "Students who had not studied two-slit interference tended to draw the diagram of intensity versus distance in part (c) as linearly decreasing or increasing" (p. 4).
- 2005 B6 was a thermodynamics question concerning an ideal gas in a cylinder. Students were given a set of data and asked to find the number of moles of gas in the cylinder after finding a relationship that could be plotted. From the Q&A: "In part (b) students showed poor graphing technique when they scaled the graph, so the data were compressed into a small region of the grid. Students also did a poor job of scaling the axes by including the origin. . . . In part (c) many students did not use the slope of the graph to obtain a value for *n* and instead simply pulled a single point from the graph or the data table" (p. 5).

The Student Performance Q&A for the AP Physics C: Mechanics Exam[3] points out several problems:

- "The salient point that comes out of the 2005 Physics C: Mechanics Exam is that students need to work on their graphing skills. It is not clear if the lack of these skills results from not handling data in a laboratory setting or from excessive reliance on software packages that do graphing for them. What is clear is that many students are unable to perform tasks involving the presentation of one-dimensional motion in a graphical form, or to analyze a set of data for orbital motion in order to extract physically significant information from it" (p. 3).

[2] Student Performance Q&A: 2005 AP Physics B Free-Response Questions.

[3] Student Performance Q&A: 2005 AP Physics C: Mechanics Free-Response Questions.

- 2005 C: Mechanics question 1 asked students to sketch a graph of velocity versus time for the upward and downward parts of a ball's flight. From the Q&A: "The most glaring error was students' inability to represent physical variables graphically. . . . Students would commonly say that the time for the ball to go up was less than the time for the ball to come down and then draw a graph that contradicted that assertion" (p. 1).
- 2005 C: Mechanics question 2 was an orbit problem involving Saturn and its moons. Students were asked to plot a set of data for four moons that would allow them to determine the mass of Saturn. From the Q&A: "The reason that the students scored so poorly on this problem was their lack of graphing skills. Students were unable to put their data in a form that would result in a linear graph, and many of those who did draw a graph were unable to use its slope to determine the mass of Saturn" (p. 2).

The Student Performance Q&A for the 2005 AP Physics C: Electricity and Magnetism Exam[4] also notes trouble spots:

- "The graph in question 3 also gave problems, even with the rather large hint given by the labeling and scaling of the graph. **Graphing skills among all the Physics C students, both in Mechanics and Electricity and Magnetism, seem to be weaker than in the past**" (p. 4, bold added for emphasis).
- C: Electricity and Magnetism question 1 asked students to consider an electric field diagram and answer questions concerning electric field, electric potential, and equipotential lines, among others. From the Q&A: "students often conflated the notion of electric field strength and potential. . . . The final difficulties centered on drawing the equipotential lines in part (d). . . . Many students failed to properly draw the equipotential line perpendicular to the field lines at the point where they intersected" (p. 2).
- C: Electricity and Magnetism question 2 was a circuit analysis problem. Part (d) of the problem asked students to sketch a graph of the current through the battery as a function of time. From the Q&A: "For those students who did know what the inductor was, the graph represented little difficulty" (p. 3).
- C: Electricity and Magnetism question 3 asked that students analyze a magnetic field problem numerically and graphically. From the Q&A: "Students had difficulty using the graph to obtain a value of μ_0. Some assumed that the slope, which is equal to $\mu_0 I$, was equal to μ_0. Others eschewed the help given to them by the labels and scale on the graph and relabeled and rescaled it" (p. 4).

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[4] Student Performance Q&A: 2005 AP Physics C: Electricity and Magnetism Free-Response Questions.

Special Focus: Graphical Analysis

These examples of the problems that students have with graphical analysis are just a subset of those that have been identified over the past several years on the AP Physics Exams. These materials will highlight some of these same areas as well as look at other areas where graphical analysis is important and necessary for student understanding of physics.

Every major topic studied in physics can and should involve the use of graphs. By using graphs frequently in class, teachers can expect students to develop familiarity and comfort with them as the course progresses. Since there are so many aspects of graphical analysis to be learned, it is probably best not to introduce them all at once but rather to introduce specific techniques when appropriate. Graphing calculators and computer graphing programs, if available, can be used as tools to quickly plot data and functions. They allow students to experiment with ideas more quickly than by plotting graphs by hand. If used judiciously, graphing calculators and software can enhance student learning. However, it is important for students to show their understanding of the graphing process and be able to plot data manually (as they may be expected to do on the AP Physics Exams).

Topics Covered in This Collection

These materials cover topics that occur in both the AP Physics B course and the C courses: kinematics, energy in mechanical and oscillatory systems, and electric field and potential. The materials also discuss topics included in only AP Physics B: P-V diagrams in thermodynamics and energy diagrams and the photoelectric effect in modern physics.

Graphical Analysis of Motion: Kinematics

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Graphical analysis is one of the most fundamental skills that introductory physics students should acquire. This article presents a practical approach that stresses conceptual understanding and interpretation of motion graphs in one dimension.

Randall Knight[1] reports that even though nearly all students can graph a set of data or can read a value from a graph, they experience difficulties with **interpreting** the information presented graphically. Some student difficulties include:

- Many students don't know the meaning of "graph *a* versus *b*." They graph the first quantity on the horizontal axis, ending up with the two axes reversed.
- Many students think that the slope of a straight-line graph is found from y/x (using any point on the graph) rather than $\Delta y/\Delta x$.
- Students don't recognize that a slope has **units** or don't know how to determine those units.
- Students don't recognize that an "area under the curve" has **units** or don't understand how the units of an "area" can be something other than area units.

Describing Motion

The study of one-dimensional kinematics is concerned with the multiple means by which the motion of objects can be represented. Such means include the use of words, graphs, equations, and diagrams.

A suggested sequence for the introduction of one-dimensional kinematics includes:

- Constant velocity: Qualitative and quantitative analysis and interpreting graphs
- Accelerated motion: Qualitative and quantitative analysis and interpreting graphs

Analysis of motion, both qualitative and quantitative, requires the establishment of a frame of reference. The exercises in this article assume a frame of reference with respect to the Earth.

The direction of motion is determined by using a Cartesian coordinate system, where

[1] R. Knight, *Instructor's Guide: Physics for Scientists and Engineers*, Prentice Hall, 2004.

the initial position is denoted as $x_0 = 0$. If the object moves to the right, its direction is positive; if it moves to the left, its direction is negative.

Arons[2] suggests that an effective way of reaching students and improving their conceptual understanding is to lead them through direct kinesthetic experiences, giving them problems in which they must translate:

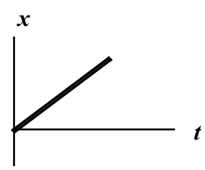
- From the graph to an actual motion
- From an actual motion to its representation on a graph

Constant Velocity: Position vs. Time Graphs

A simple analysis of constant velocity can be done using a bowling ball rolling on a carpeted floor or using a battery-operated car. Video analysis is a great tool for analyzing the motion in detail. The objective is for the students to be able to interpret graphs of *x* vs. *t* in different directions. Here are some examples:

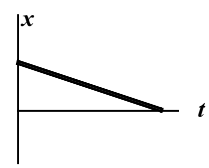
Give a qualitative description of the motion depicted in the following *x* vs. *t* graphs:

a.



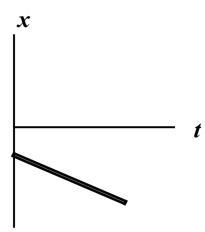
Solution: Object starts at the origin and moves in the positive direction with constant velocity.

b.



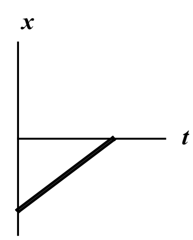
Solution: Object starts to the right of the origin and moves in the negative direction with constant velocity ending at the origin.

c.



Solution: Object starts at the origin and moves in the positive direction with constant velocity.

d.



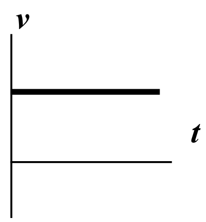
Solution: Object starts to the right of the origin and moves in the negative direction with constant velocity ending at the origin.

[2] A. B. Arons, *Teaching Introductory Physics*, John Wiley & Sons, 1997.

Constant Velocity: Velocity vs. Time Graphs

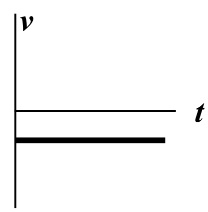
Give a qualitative description of the motion depicted in the following *v* vs. *t* graphs:

a.



Solution: Object moves to the right at a fast constant speed.

b.



Solution: Object moves to the left at a slow constant speed.

Quantitative Approach

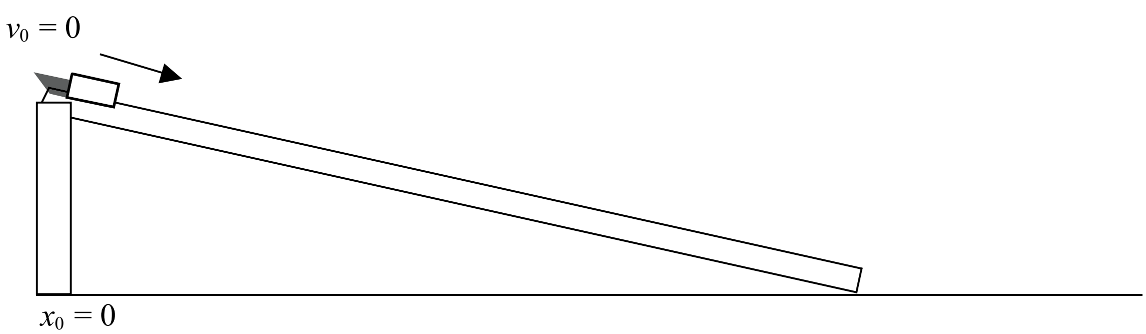
The next step is to have the students calculate the slope of an *x* vs. *t* graph and understand that the value obtained is the average velocity. When the velocity is constant, the average velocity over any time interval is equal to the instantaneous velocity at any time.

The students should also be able to calculate the area under the curve of a *v* vs. *t* graph and understand that the value obtained is the displacement.

Accelerated Motion

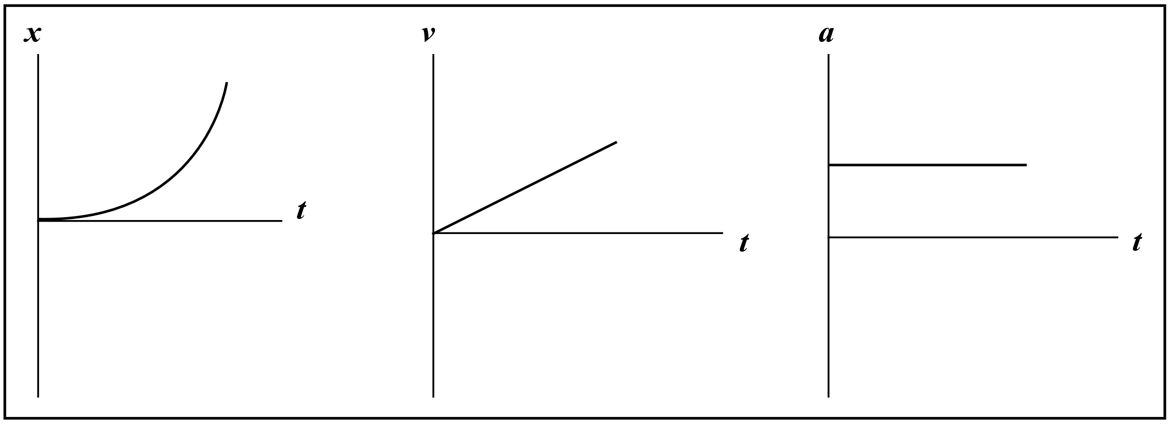
McDermott[3] and her Physics Education Research group have suggested an excellent approach that presents students with situations of a ball rolling along a series of level and inclined tracks. This experiment can be performed in the classroom or lab using a ball-track setup or a dynamics track and a cart.

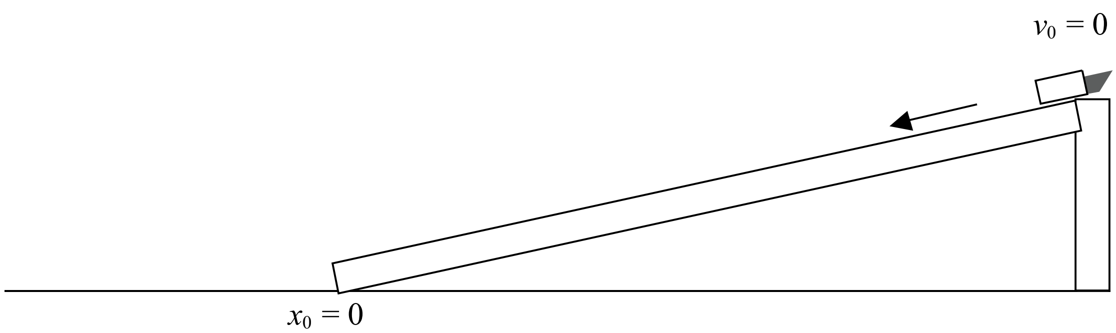
The students should draw qualitative graphs of *x* vs. *t*, *v* vs. *t*, and *a* vs. *t*.



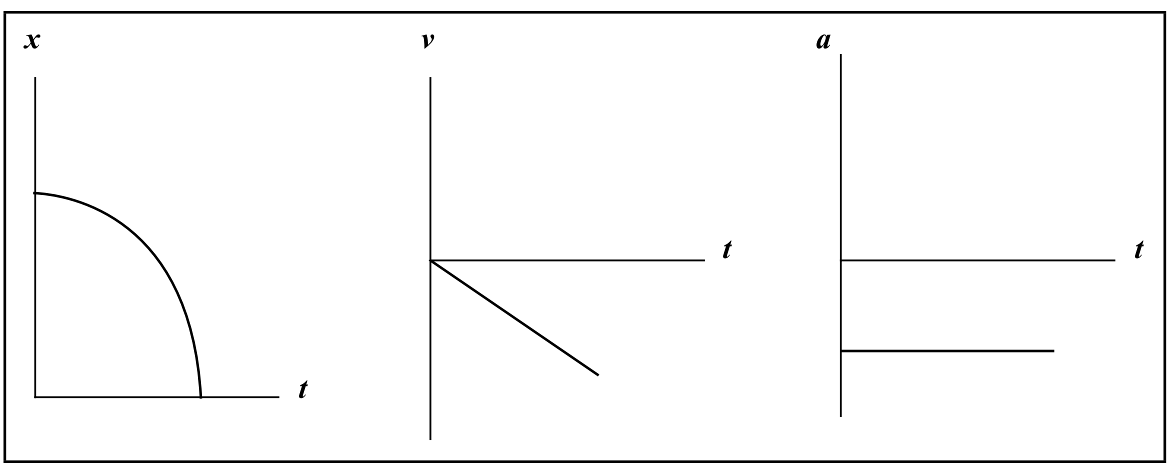
[3] L. C. McDermott and P. S. Shaffer, *Tutorials in Introductory Physics*, Prentice Hall, 2002.

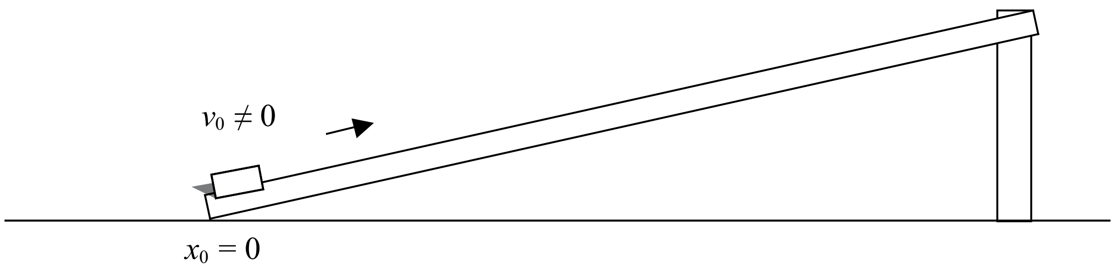
Solution:



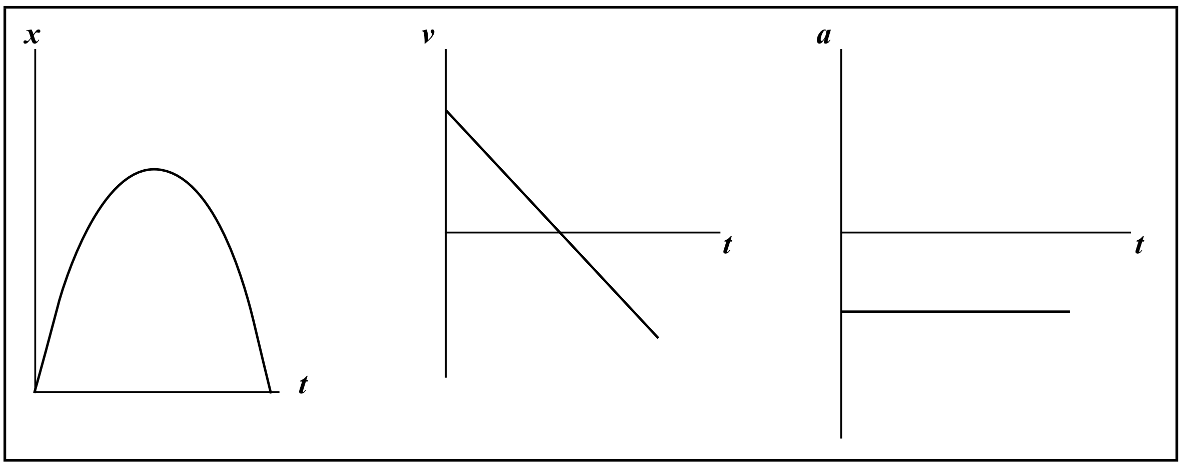


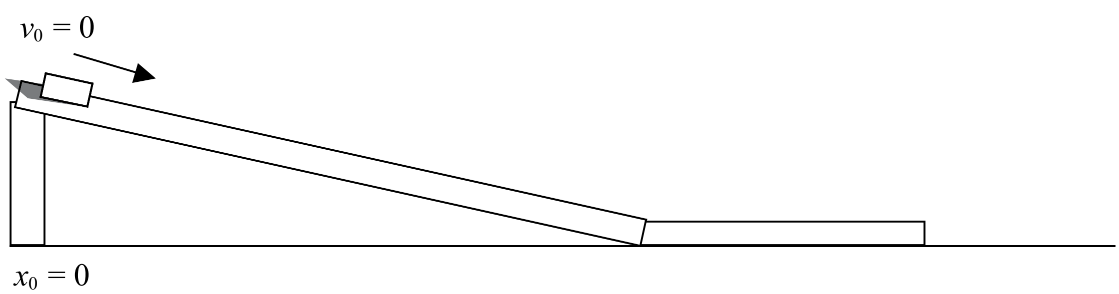
Solution:



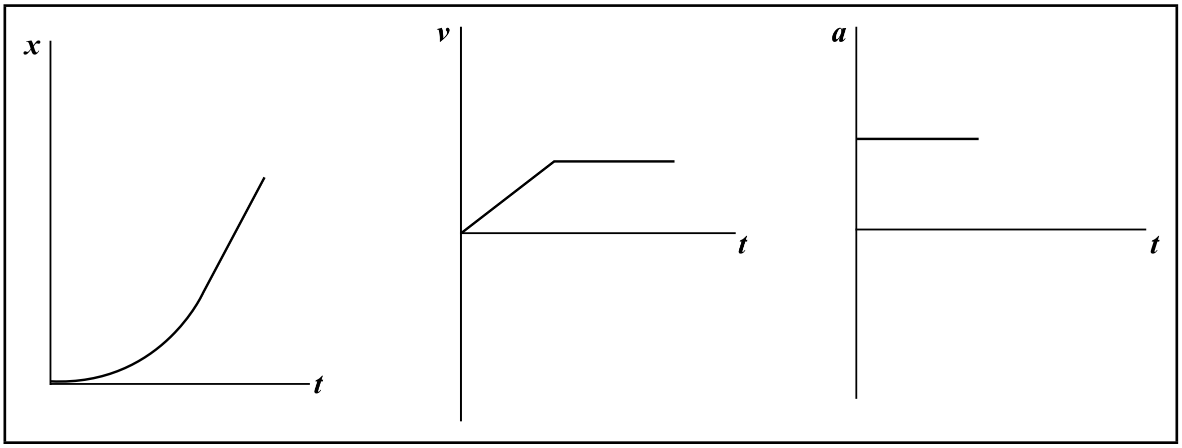


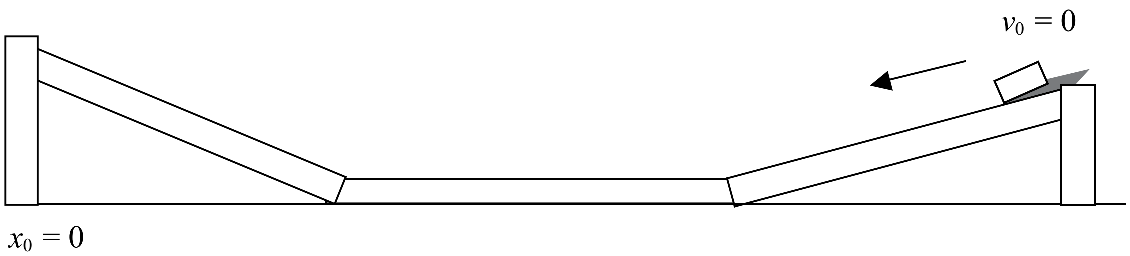
Solution:



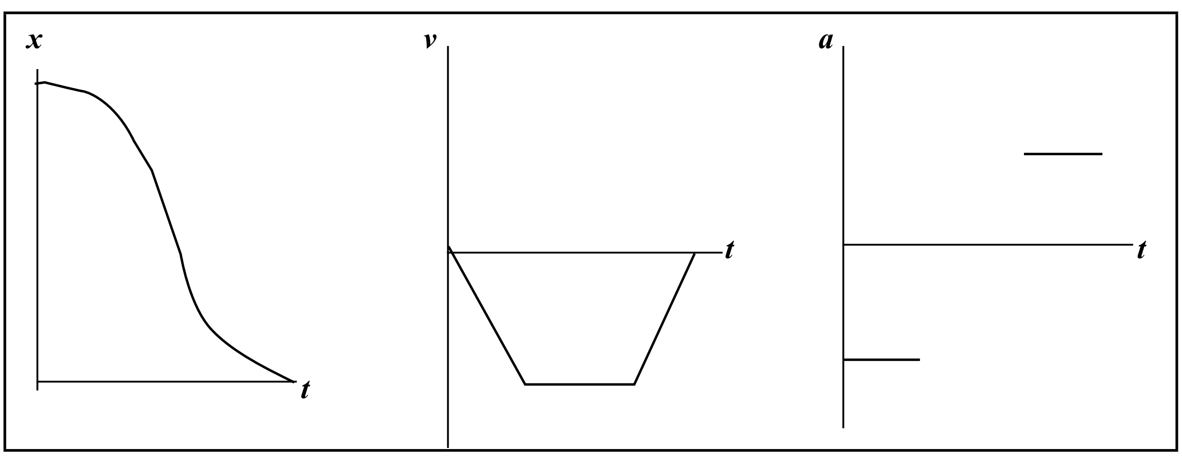


Solution:





Solution:

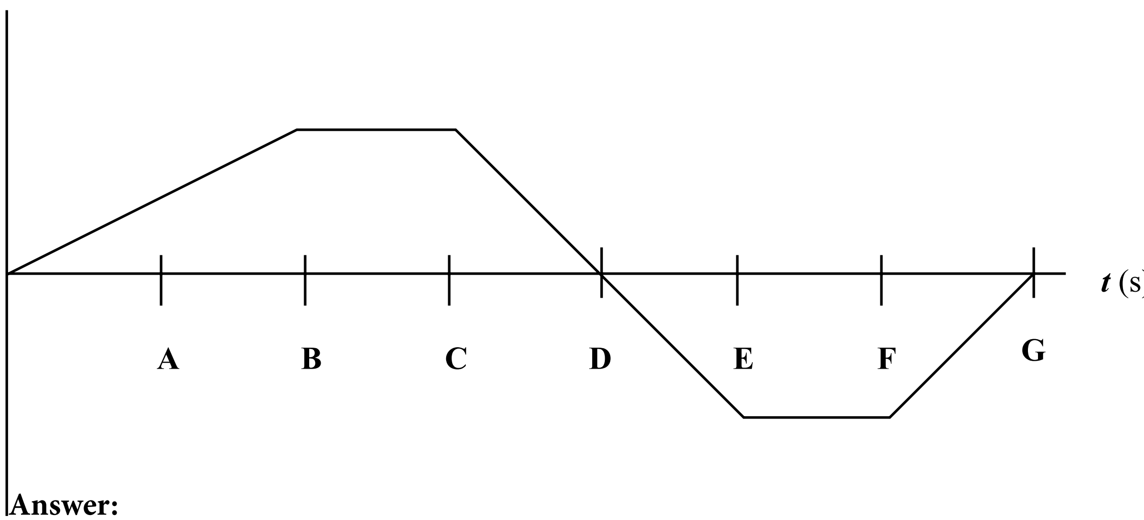


After the students have made their predictions, they should conduct different experiments to verify their graphs. The use of motion detectors and software programs such as *Logger Pro*™ or *Graphical Analysis*™ is very effective in this analysis.

This qualitative approach will help the students understand that the signs of the velocity and the acceleration are the same if the object is **speeding up** and that the signs of the velocity and the acceleration are the opposite if the object is **slowing down**.

Interpreting Graphs

1. Give a qualitative description of the motion of an object at the different time intervals depicted in the following v versus t graph:



Answer:

A-B Positive acceleration, object is speeding up

B-C Object is moving with positive constant velocity

C-D Negative acceleration, object is slowing down

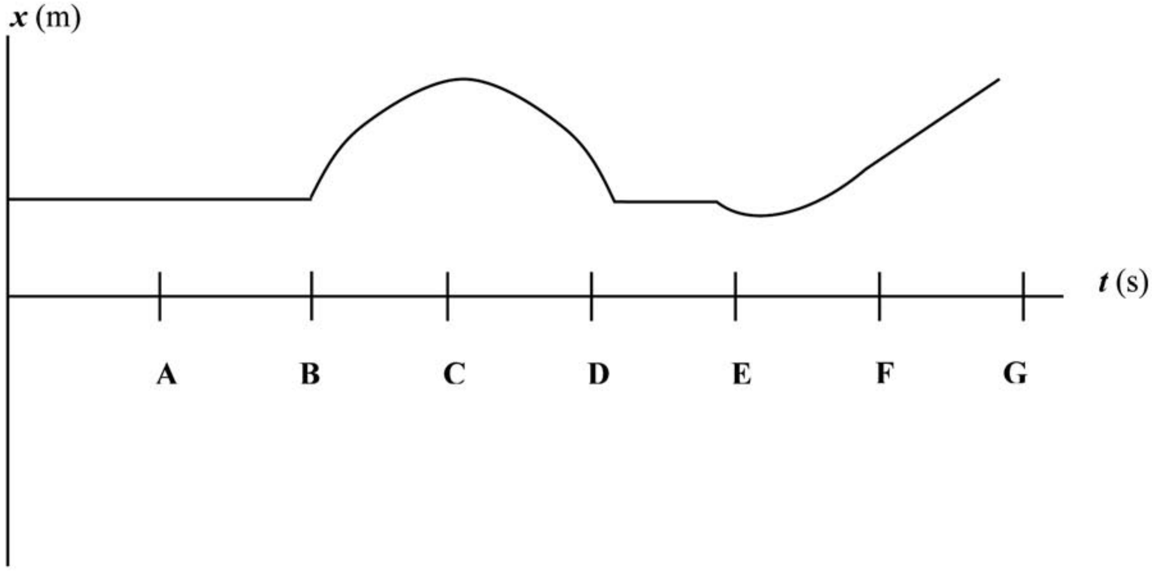
D-E Negative acceleration, object is speeding up

E-F Object is moving with negative constant velocity

F-G Positive acceleration, object is slowing down

Special Focus: Graphical Analysis

2. Give a qualitative description of the motion of an object at the different time intervals depicted in the following x versus t graph:



Answer:

A-B Object is at rest

B-C Negative acceleration, object is slowing down

C-D Negative acceleration, object is speeding up

D-E Object is at rest

E-F Positive acceleration, object is speeding up

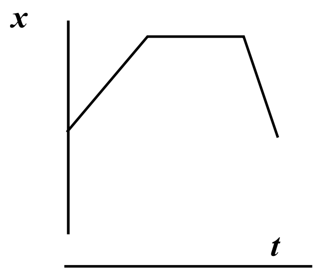
F-G Object is moving with positive constant velocity

Quantitative Approach

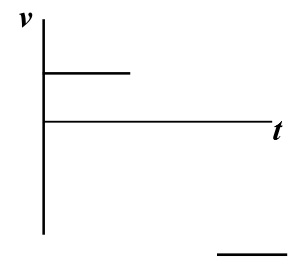
The quantitative approach should include calculations of:

- Slope of the tangent of an x vs. t graph and definition of instantaneous velocity
- Slope of the v vs. t graph and the understanding that the value obtained is the average acceleration
- Area under the v vs. t graph and the understanding that it gives the displacement
- Area under the a vs. t graph and the understanding that it gives the change in velocity

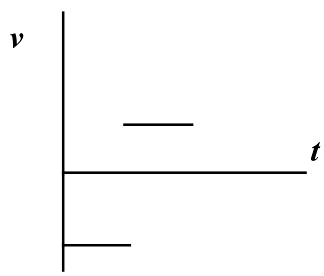
1.



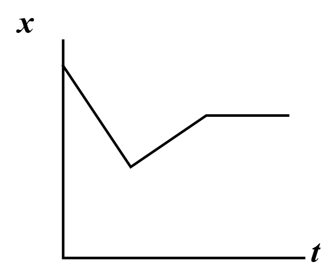
Solution:
Object starts at the right of the origin, moves right at constant *v*, stands still, and then moves left at faster constant *v*.



2.



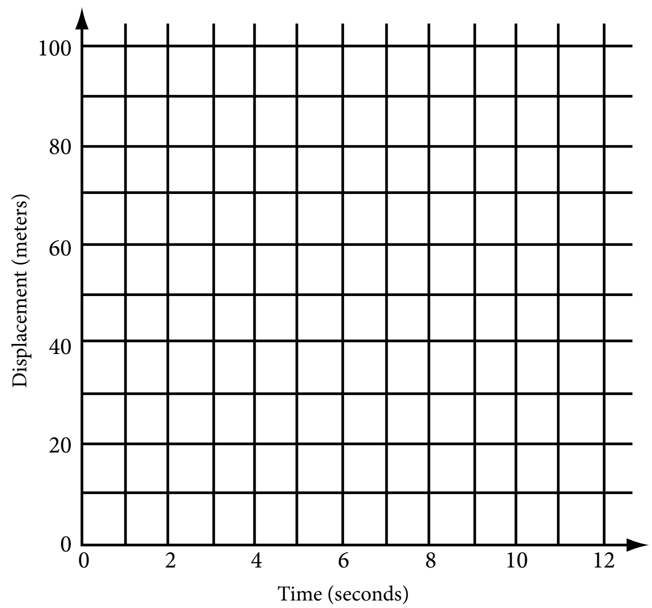
Solution:
Object moves left at constant *v*, then moves to the right at constant *v* but slower, then stands still.

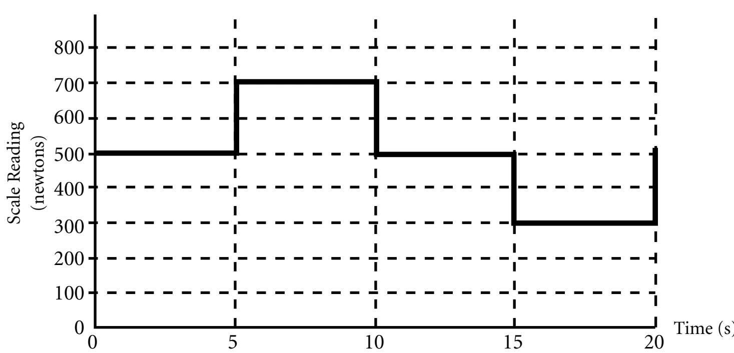


Graphical Analysis of Motion: Free-Response Questions from Past AP Physics Exams

Answers to these questions can be found in College Board publications, on the AP Central Web site, or at AP Summer Institutes and workshops.

1982 Physics B, Question 1
The first meters of a 100 meter dash are covered in 2 seconds by a sprinter who starts from rest and accelerates with a constant acceleration. The remaining 90 meters are run with the same velocity the sprinter had after 2 seconds.
a. Determine the sprinter’s constant acceleration during the first 2 seconds.
b. Determine the sprinter’s velocity after 2 seconds have elapsed.
c. Determine the total time needed to run the full 100 meters.
d. On the axes provided below, draw the displacement vs. time curve for the sprinter.





1993 Physics B, Question 1

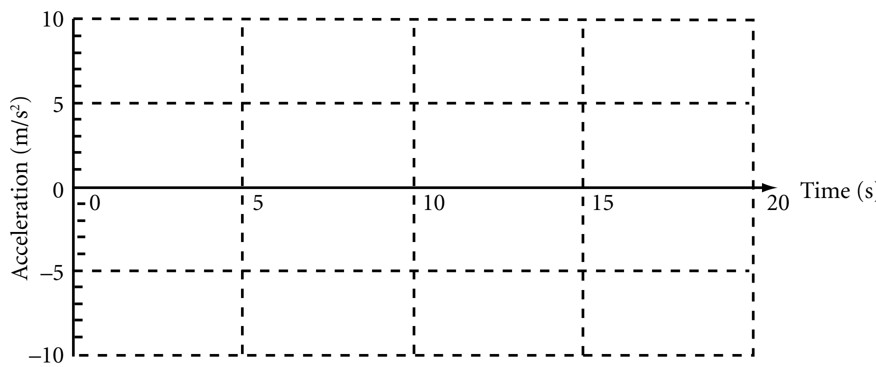
A student whose normal weight is 500 newtons stands on a scale in an elevator and records the scale reading as a function of time. The data are shown in the graph above. At time $t = 0$, the elevator is at displacement $x = 0$ with velocity $v = 0$. Assume that the positive directions for displacement, velocity, and acceleration are upward.

a. On the diagram below, draw and label all of the forces on the student at $t = 8$ seconds.

b. Calculate the acceleration a of the elevator for each 5 second interval.
   i. Indicate your results by completing the following table.

Time Interval (s)	0–5	5–10	10–15	15–20
a (m/s²)	____	____	___	____

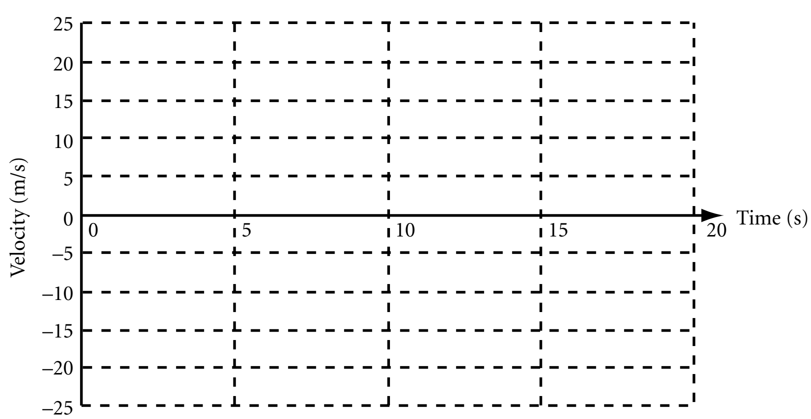
ii. Plot the acceleration as a function of time on the following graph.



c. Determine the velocity v of the elevator at the end of each 5 second interval.
   i. Indicate your results by completing the following table.

Time Interval (s)	0–5	5–10	10–15	15–20
v (m/s)	____	____	___	____

ii. Plot the velocity as a function of time on the following graph.

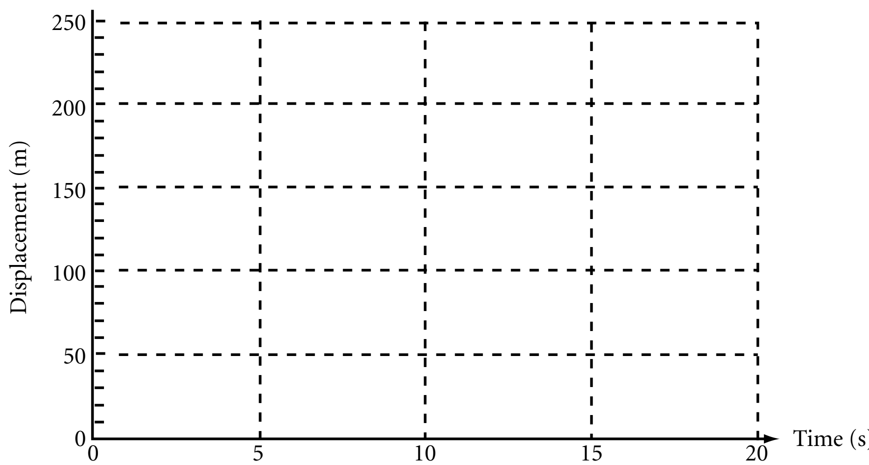


d. Determine the displacement *x* of the elevator above the starting point at the end of each 5 second interval.

i. Indicate your results by completing the following table.

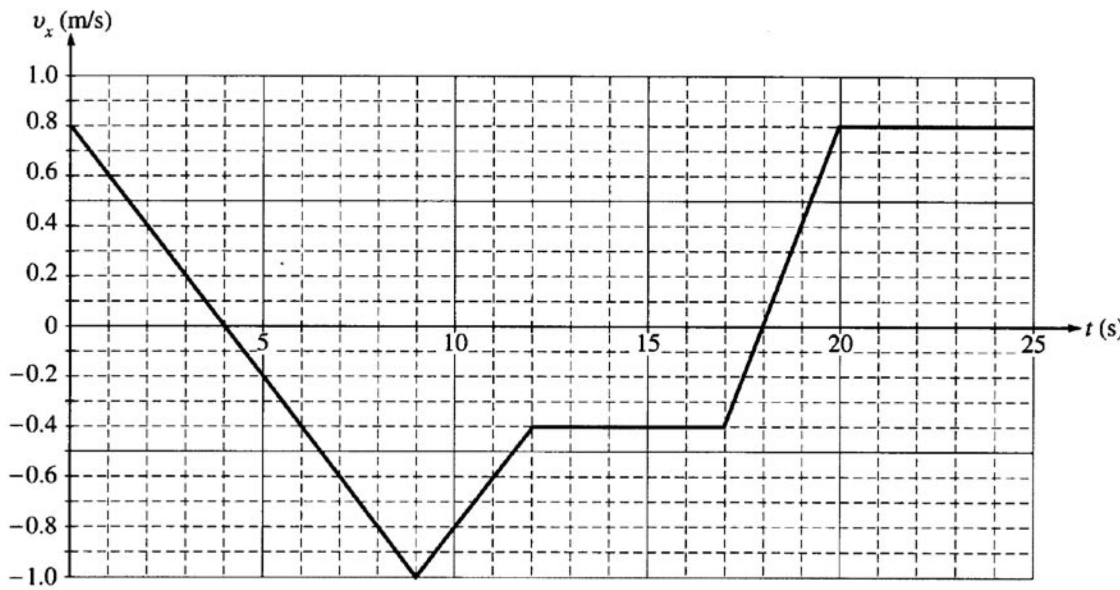
Time Interval (s)	0–5	5–10	10–15	15–20
x (m)	____	____	___	____

ii. Plot the displacement as a function of time on the following graph.



2000 Physics B, Question 1

A 0.50 kg cart moves on a straight horizontal track. The graph of velocity *v* versus time *t* for the cart is given below.

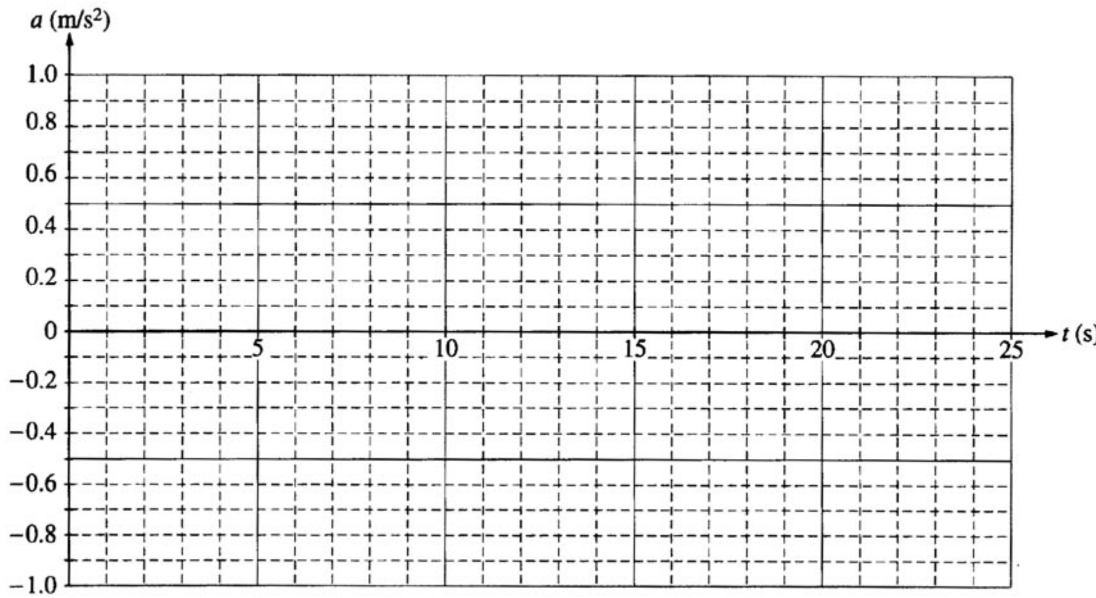


a. Indicate every time *t* for which the cart is at rest.

b. Indicate every time interval for which the speed (magnitude of velocity) of the cart is increasing.

c. Determine the horizontal position *x* of the cart at *t* = 9.0 s if the cart is located at x = 2.0 m at *t* = 0.

d. On the axes below, sketch the acceleration *a* versus time *t* graph for the motion of the cart from $t = 0$ to $t = 25$ s.



e. From $t = 25$ s until the cart reaches the end of the track, the cart continues with constant horizontal velocity. The cart leaves the end of the track and hits the floor, which is 0.40 m below the track. Neglecting air resistance, determine each of the following:

i. The time from when the cart leaves the track until it first hits the floor

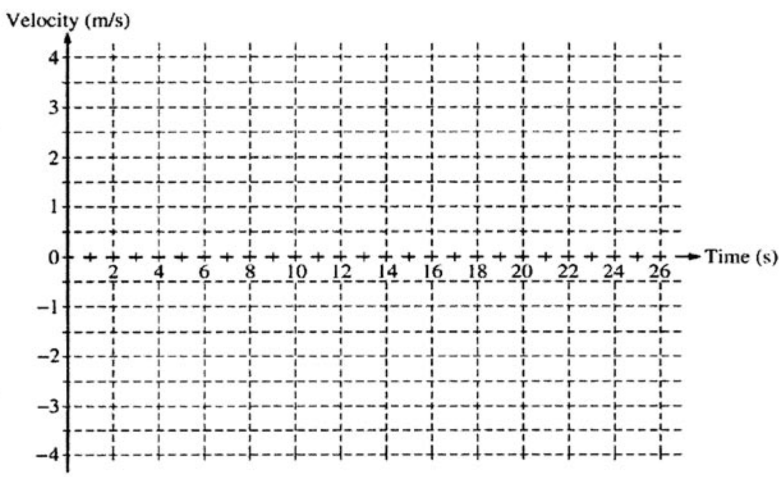
ii. The horizontal distance from the end of the track to the point at which the cart first hits the floor

iii. The kinetic energy of the cart immediately before it hits the floor

2005 Physics B, Question 1

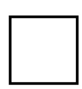
The vertical position of an elevator as a function of time is shown above.

a. On the grid below, graph the velocity of the elevator as a function of time.



b. i. Calculate the average acceleration for the time period $t = 8$ s to $t = 10$ s.

ii. On the box below that represents the elevator, draw a vector to represent the direction of this average acceleration.



c. Suppose that there is a passenger of mass 70 kg in the elevator. Calculate the apparent weight of the passenger at time $t = 4$ s.

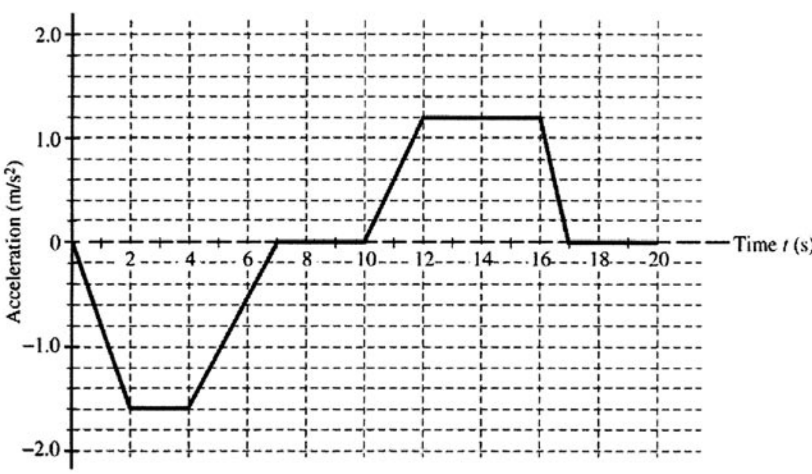
Special Focus: Graphical Analysis

2005 Physics B, Form B, Question 1

A student of mass m stands on a platform scale in an elevator in a tall building. The positive direction for all vector quantities is upward.

a. Draw a free body diagram showing and labeling all the forces acting on the student, who is represented by the dot below.

b. Derive an expression for the reading on the scale in terms of the acceleration a of the elevator, the mass m of the student, and fundamental constants. An inspector provides the student with the following graph showing the acceleration a of the elevator as a function of time t.



c. i. During what time interval(s) is the force exerted by the platform scale on the student a maximum value?

ii. Calculate the magnitude of that maximum force for a 45 kg student.

d. During what time interval(s) is the speed of the elevator constant?

Energy Diagrams in Mechanical Systems and the Graphs for Oscillatory Systems

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A variety of activities for graphical analysis pertaining to energy in mechanical and oscillatory systems are presented below. This material, divided into sections I through VI, will assist AP Physics teachers in helping students sharpen their analytical skills.

Energy in Mechanical Systems

A mechanical system consists of one or more particles or rigid bodies. These objects may interact:

- With each other
- With a field, such as a gravitational, electrical, or magnetic field
- With a spring

Kinetic Energy		
Form	**Symbol**	**Formula**
Translational kinetic energy	K_{TRANS}	$\frac{1}{2}mv^2$
Rotational kinetic energy	K_{ROT}	$\frac{1}{2}\omega^2$
Vibrational kinetic energy in SHM (simple harmonic motion)	K_{SHM}	$\frac{1}{2}\omega^2\left(x_0^2 - x^2\right)$

Special Focus: Graphical Analysis

Potential Energy		
Form	**Symbol**	**Formula**
Gravitational potential energy	U_g	mgh
Elastic potential energy	U_s	$\frac{1}{2}kx^2$
Electric potential energy	U_e	$\frac{k_e q_1 q_2}{r}$

Different Kinds of Graphs

The graph of an equation containing variables x and y can be linear or nonlinear depending on:

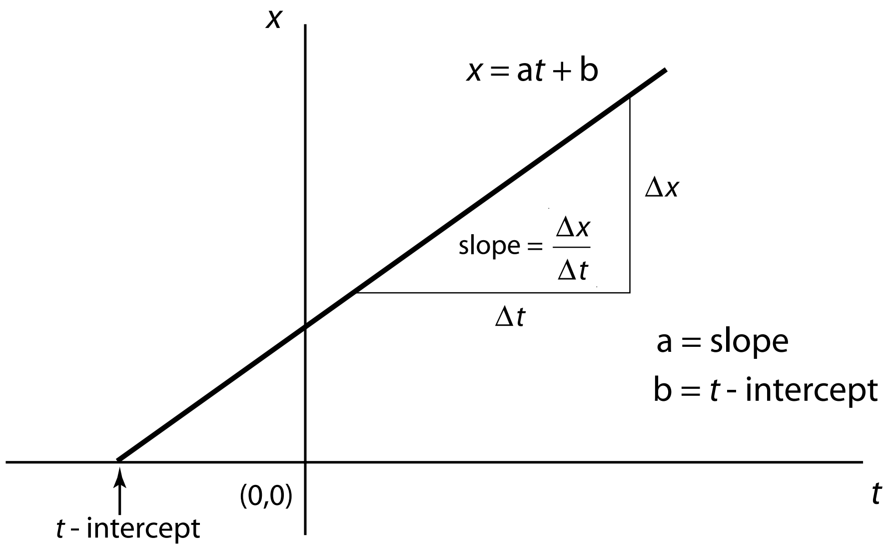
- The nature of the equation, i.e., linear, quadratic, exponential, and so on
- The quantities that are plotted, for example, y vs. x , y vs. x^2 or y^2 vs. x^2, y vs. $\sqrt{x}$

A nonlinear equation can yield a linear graph if appropriate variables are used for the two axes.

Various quantities associated with the graph, such as slope and x- and y-intercepts, reveal more information about the relationship between the variables.

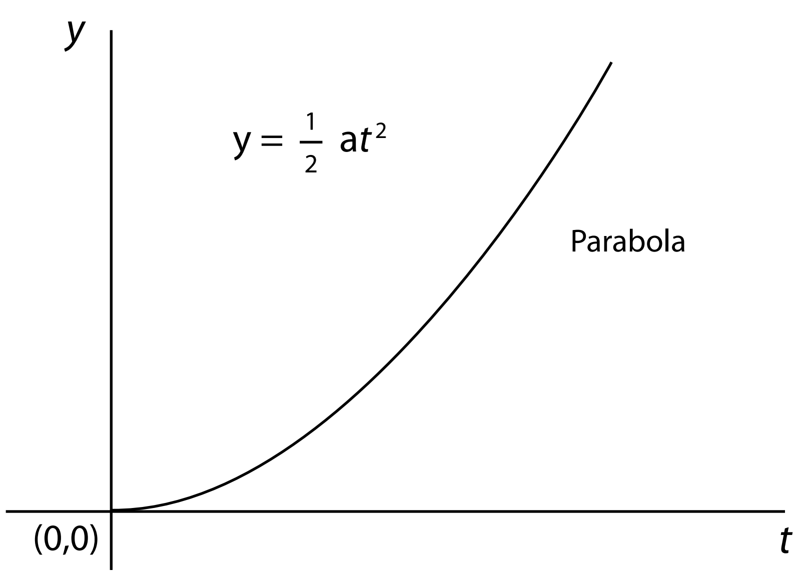
Examples

1. For a linear equation $x = at + b$, the graph of x vs. t would be a straight line. The slope of the straight line will give the value of the constant **a**, and the x-intercept will yield the value of the second constant **b**.

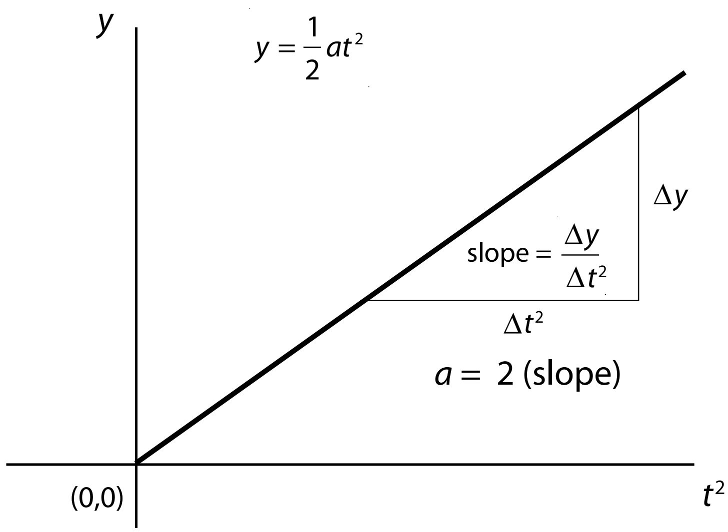


2. For a nonlinear equation $y = \frac{1}{2}at^2$:

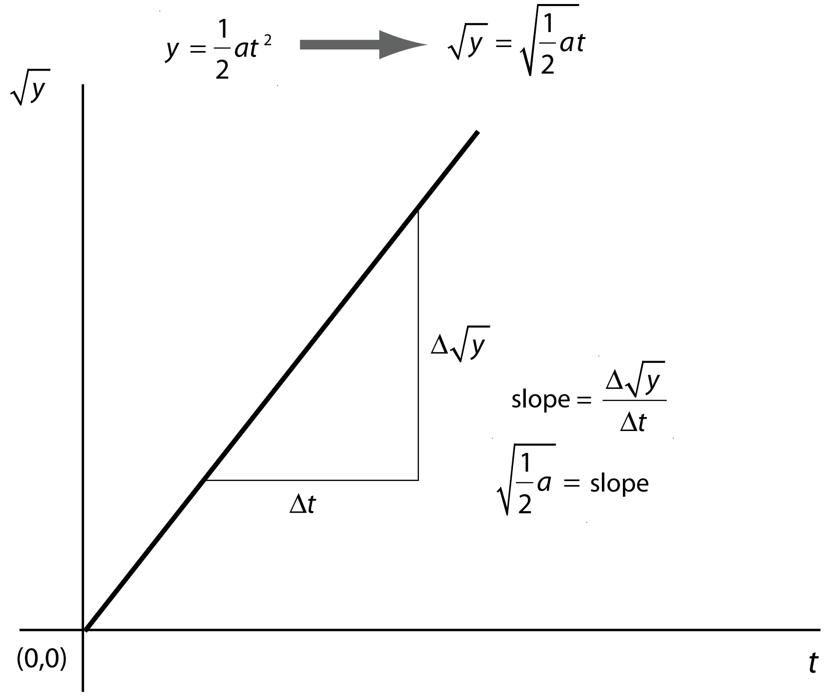
   a. The graph of y vs. t will yield parabola. However, the constant "**a**" cannot be obtained easily from this nonlinear graph unless a curve-fitting program is used.



b. The graph of y vs. t^2 will yield a straight line with slope = $\frac{1}{2}a$; hence the constant **a** can be readily calculated from the slope of the straight-line graph.



c. The graph of $\sqrt{y}$ vs. t will also yield a straight line with slope = $\sqrt{\frac{1}{2}a}$.

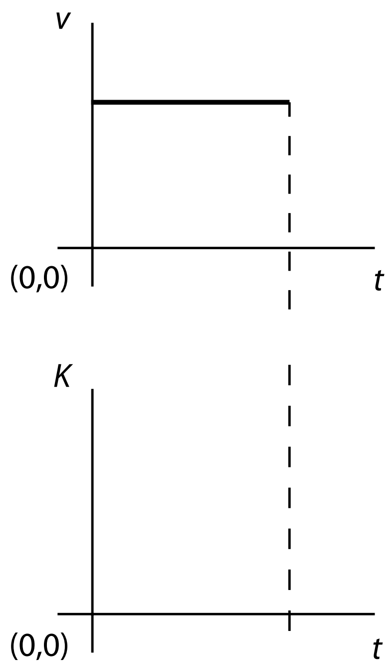


One can argue that choice b would be the most convenient one.

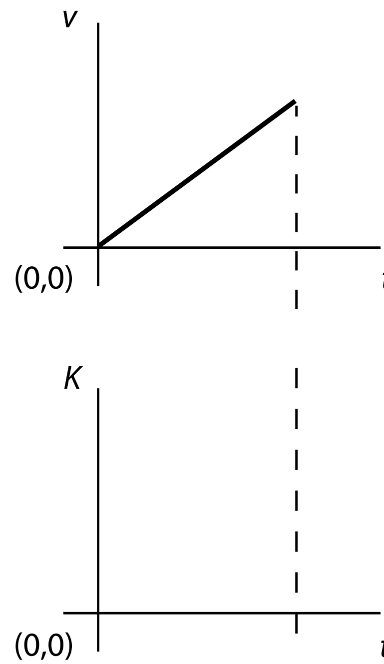
Problems

I. Each of the following graphs represents a *v* vs. *t* relationship for a particle moving along a straight line. Sketch the corresponding graph of *K* vs. *t*.

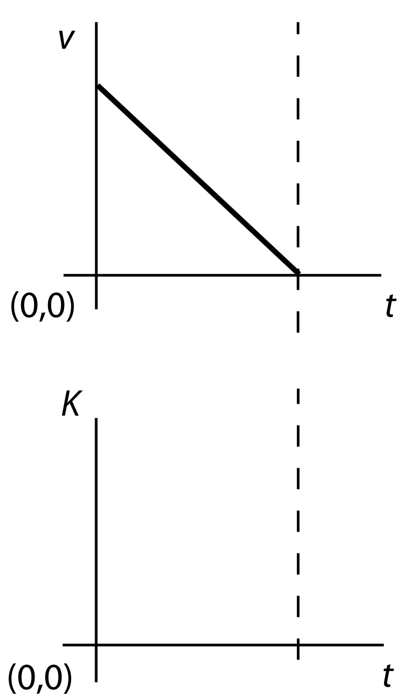
1.



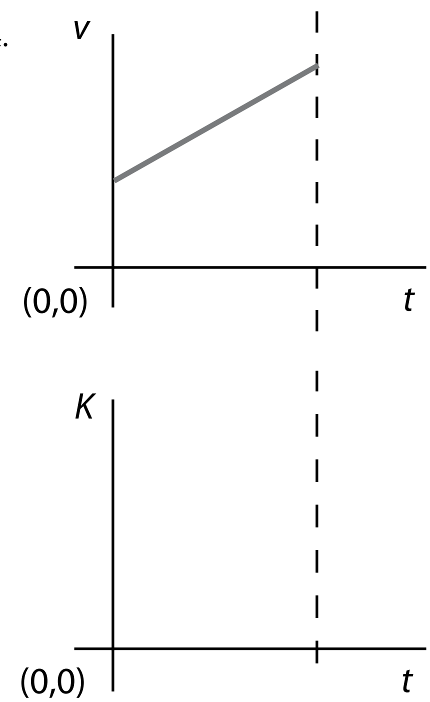
2.



3.

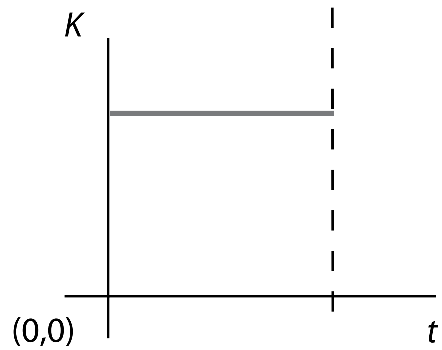


4.

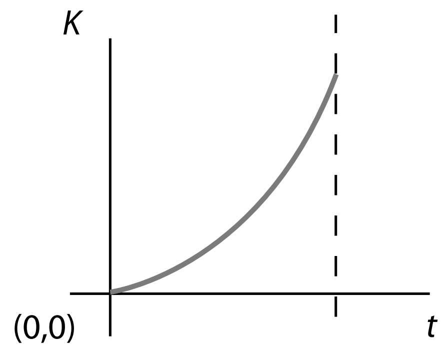


Solutions:

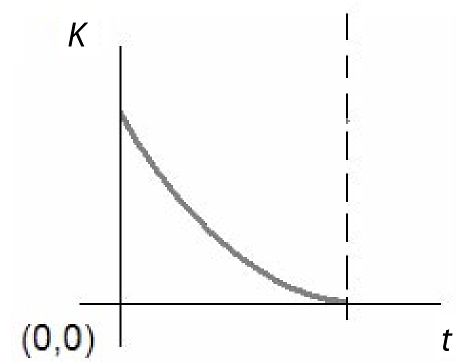
1. v is constant with t in the given graph, hence $K = \frac{1}{2}mv^2$ is also constant with t.



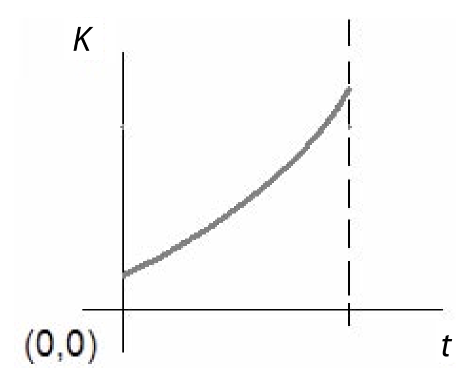
2. v is increasing linearly with t, hence $K = \frac{1}{2}mv^2$ is proportional to t^2. v is 0 at $t = 0$, hence the graph passes through the origin. The slope is 0 at the origin, and it is increasing with t. The graph is a parabola.



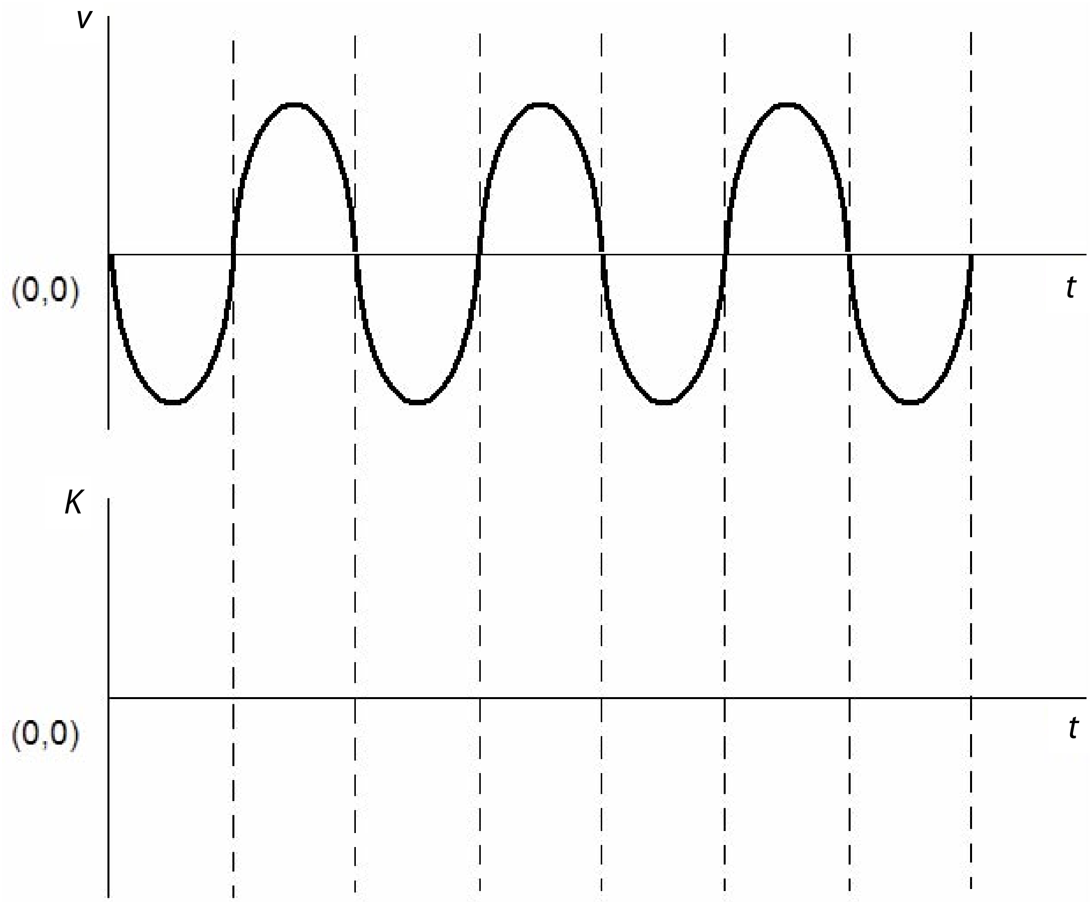
3. Again, the graph is a parabola. $v > 0$ at $t = 0$, hence $K > 0$ at $t = 0$. v is decreasing, hence the slope of K vs. t is negative. $v = 0$ at the end, hence the slope of K vs. t is zero at that point.



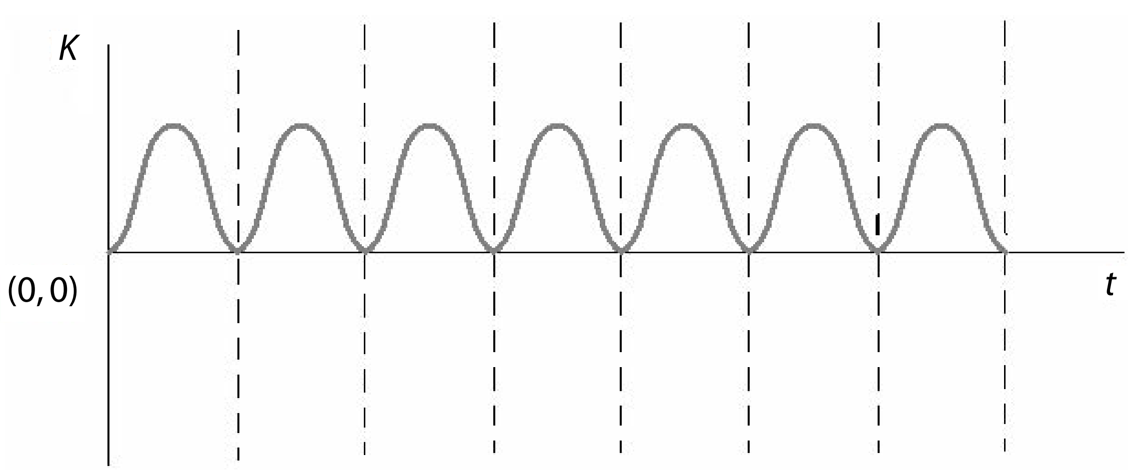
4. Similar to no. 2, but $v > 0$ at $t = 0$, hence $K > 0$ at $t = 0$.



II. Below is the *v* vs. *t* graph for a particle *m*, undergoing SHM (simple harmonic motion). Sketch the corresponding graph of *K* vs. *t* of the particle.

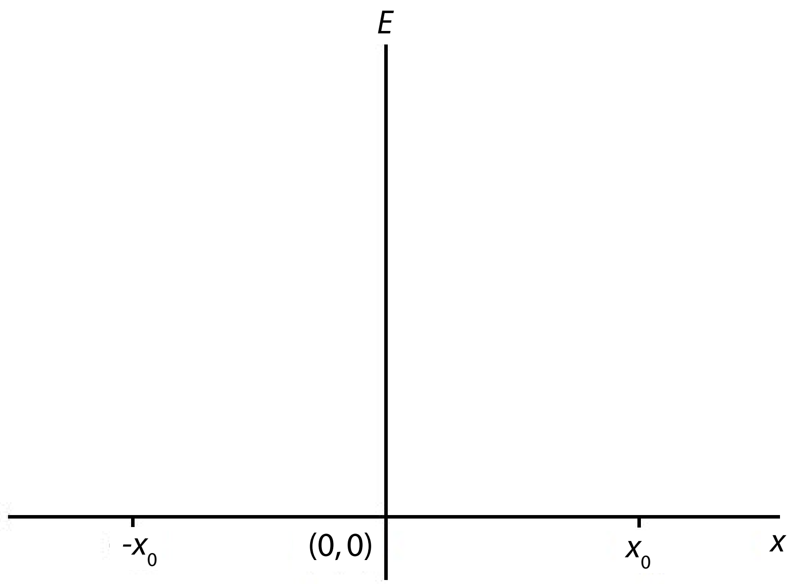


Solution:

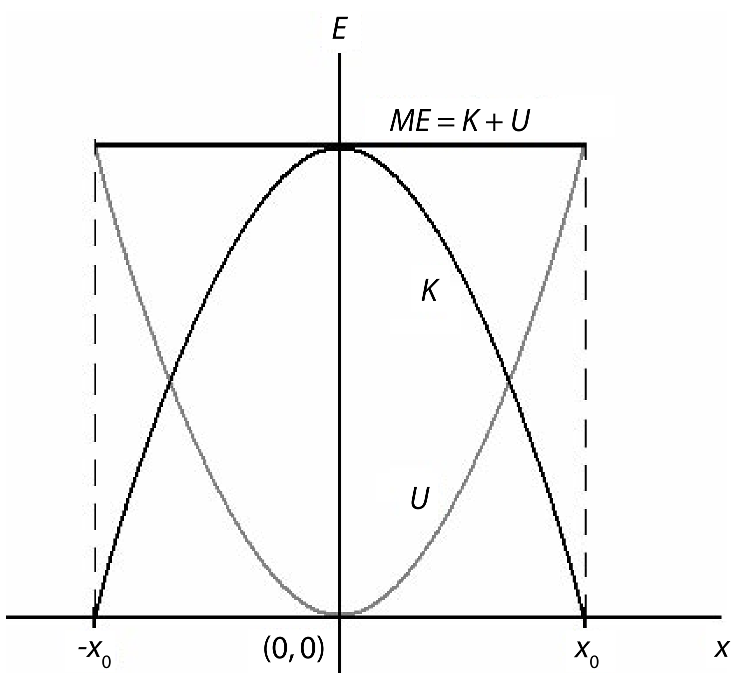


III. A particle is undergoing undamped SHM about $x = 0$ with amplitude x_0. K, U, and ME represent the kinetic energy, potential energy, and total mechanical energy of the particle.

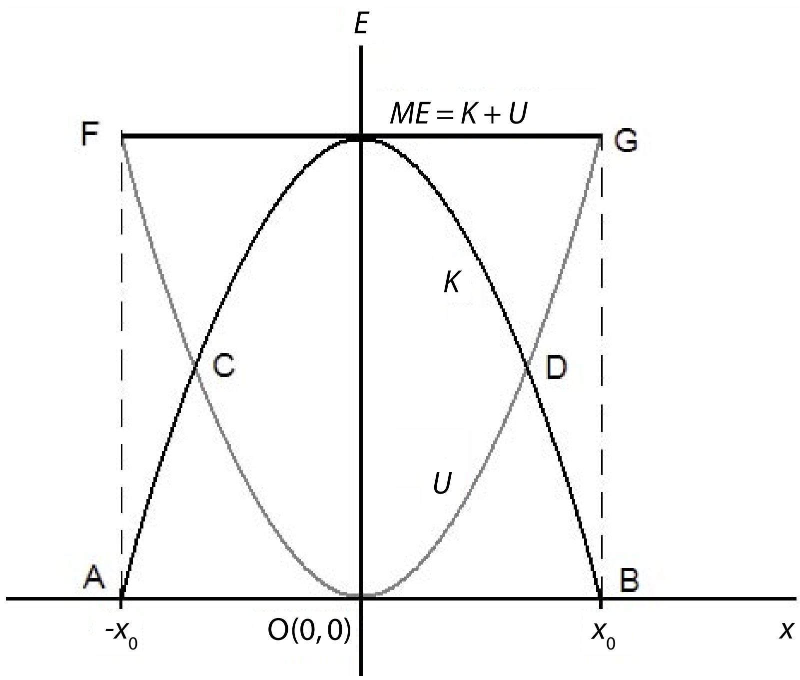
On the same coordinate system below, sketch graphs for K vs. x, U vs. x, and ME vs. x.



Solution:



IV. A particle is undergoing undamped SHM about $x = 0$ with amplitude x_0. K, U, and ME represent the kinetic energy, potential energy, and total mechanical energy of the particle. Below are the graphs of the three energies vs. x on the same coordinate system:



1. At what point(s) is the K of the system minimum?
   a. At x_0 and $-x_0$
   b. At x_0 only
   c. At $-x_0$ only
   d. At O only
   e. None; it is constant.

2. At what point(s) is the U of the system minimum?
   a. At x_0 and $-x_0$
   b. At x_0 only
   c. At $-x_0$ only
   d. At O only
   e. None; it is constant.

3. At what point(s) is the *ME* of the system minimum?
   a. At x_0 and $-x_0$
   b. At x_0 only
   c. At $-x_0$ only
   d. At O only
   e. None; it is constant.

4. At what point(s) is the *K* of the system maximum?
   a. At x_0 and $-x_0$
   b. At x_0 only
   c. At $-x_0$ only
   d. At O only
   e. None; it is constant.

5. At what point(s) is the *U* of the system maximum?
   a. At x_0 and $-x_0$
   b. At x_0 only
   c. At $-x_0$ only
   d. At O only
   e. None; it is constant.

6. At what point is $K = U$ for the system?
   a. At one point: somewhere between O and A only
   b. At one point: somewhere between O and B only
   c. At two points: somewhere between O and A and O and B
   d. At O only
   e. At two points: A and B

Solutions:

1. a: K is zero at x_0 and $-x_0$, positive otherwise.

2. d: U is zero at O, positive otherwise.

3. e: The *ME* vs. x graph is a horizontal line.

4. d: K decreases to zero as the particle moves to the extremes.

5. a: U decreases to zero as the particle moves toward O.

6. c: The K vs. x and U vs. x graphs intersect at two points.

V. A student performs a simple pendulum lab. The purpose of the lab is to:

a. Verify the relationship between the period T and the length l of the simple pendulum $T = 2\pi\sqrt{\frac{l}{g}}$

b. Determine the value of acceleration due to gravity g

The data collected by the student is given below:

Trial Number	Length of Simple Pendulum: l (m)	Time for 10 Oscillations: t (s)
1	0.408	13.2
2	0.508	13.8
3	0.608	16.0
4	0.708	16.5
5	0.808	18.0
6	0.908	19.0
7	1.008	20.5

1. State at least two different ways the data above can be analyzed graphically for the purpose of the lab.

Solution:
For verifying the relationship between T and l, any of the following graphs can be drawn:

- T vs. $\sqrt{l}$ and draw a best-fit straight line
- T^2 vs. l and draw a best-fit straight line
- T vs. l and draw a best-fit parabola

A good fit (low scattering of points around the curve) will indicate that the equation $T = 2\pi\sqrt{\frac{l}{g}}$ has been verified.

2. Which graph is preferable for determining g? How can g be calculated from it?

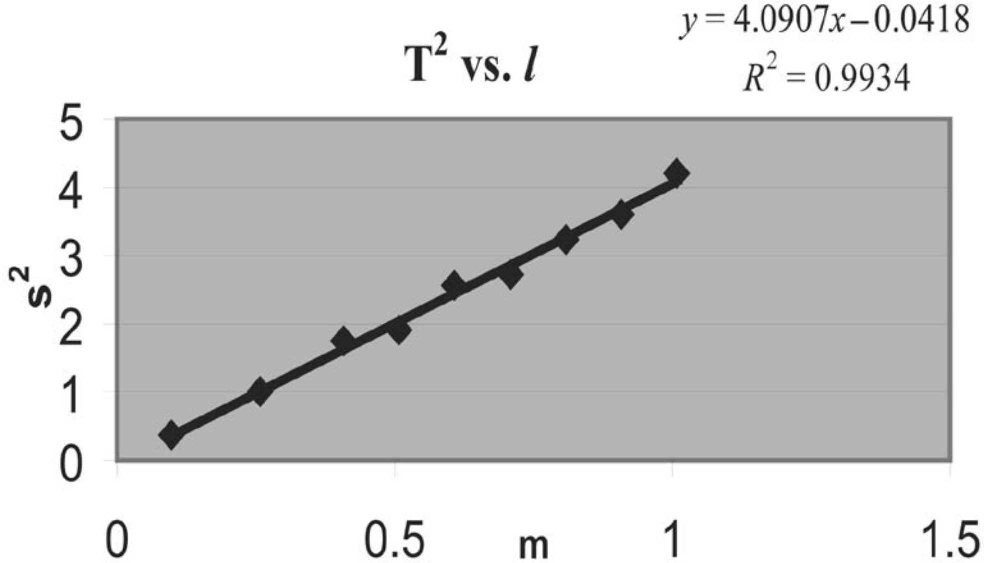
Solution:
Both straight-line graphs are preferable. The value of g can be determined directly from the slope of the straight line:

- For the T vs. $\sqrt{l}$ graph, slope = $\frac{2\pi}{\sqrt{g}} \rightarrow g = \frac{4\pi^2}{\text{slope}^2}$.
- For the T^2 vs. l graph, slope = $\frac{4\pi^2}{g} \rightarrow g = \frac{4\pi^2}{\text{slope}^2}$.

3. Add and fill more columns if necessary to draw a straight-line graph.

Trial Number	Length of Simple Pendulum: l (m)	Time for 10 Oscillations: t (s)	Time Period: $T = \frac{t}{10}$ (s)	T^2: (s^2)
1	0.408	13.2	1.32	1.74
2	0.508	13.8	1.38	1.90
3	0.608	16.0	1.60	2.56
4	0.708	16.5	1.65	2.72
5	0.808	18.0	1.80	3.24
6	0.908	19.0	1.90	3.61
7	1.008	20.5	2.05	4.20

4. Draw a best-fit straight line and calculate g from the graph.



Solution:
Slope = 4.091
$g = 4\pi^2/\text{slope} = 9.64\ \text{m/s}^2$

This graph verifies that the period T is proportional to $\sqrt{l}$.

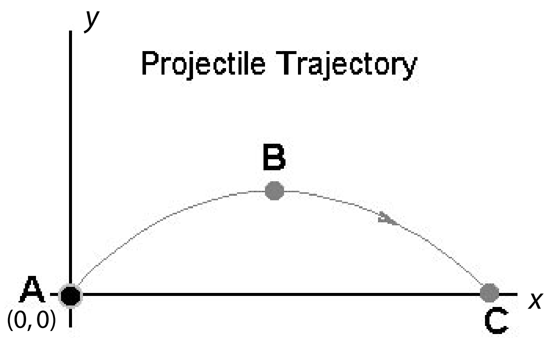
5. Calculate percent error using g = 9.81 m/s2 as the accepted value.

Solution:
Error analysis:

$$\%\ \text{Error} = \left|\frac{\text{Actual value} - \text{Experimental value}}{\text{Actual value}}\right|100$$

$$= \left|\frac{9.81 - 9.65}{9.81}\right|100 = 1.63\%$$

VI. Consider a football kicked from level ground. The ball reaches the maximum height at B and returns to the ground at C.



1. At which point(s) does the ball have maximum gravitational potential energy?
   a. At A and C
   b. At A only
   c. At C only
   d. At B only
   e. The gravitational potential energy is constant over the entire trajectory.

2. At which point(s) does the ball have maximum kinetic energy?
   a. At A and C
   b. At A only
   c. At C only
   d. At B only
   e. The kinetic energy is constant over the entire trajectory.

3. At which point(s) does the ball have minimum gravitational potential energy?
   a. At A and C
   b. At A only
   c. At C only
   d. At B only
   e. The gravitational potential energy is constant over the entire trajectory.

4. At which point(s) does the ball have minimum kinetic energy?
   a. At A and C
   b. At A only
   c. At C only
   d. At B only
   e. The kinetic energy is constant over the entire trajectory.

5. At which point(s) does the ball have zero kinetic energy?
   a. At A and C
   b. At A only
   c. At C only
   d. At B only
   e. At none of these

6. At which point(s) does the ball have maximum total mechanical energy?
   a. At A and C
   b. At A only
   c. At C only
   d. At B only
   e. The mechanical energy is constant over the entire trajectory.

Solutions:

1. d: B is the highest point.

2. a: These are the lowest points of the trajectory.

3. a: These are the lowest points of the trajectory.

4. d: B is the highest point.

5. d: The ball is never stationary but has minimum speed at B.

6. e: In the absence of air resistance, $ME = K + U$ is constant.

Special Focus: Graphical Analysis

Field and Potential Graphs

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Instructional Strategies

The point of this assignment is to stress the idea that a Coulomb force acting on a particle and the particle's potential energy are dependent, essentially, on two factors: the charge of the particle and the electric field created by **other** charged particles.

Prior to attempting this assignment, the students should be introduced to Coulomb's law and reminded about the importance of sketching free-body diagrams for each case before answering the question (which should provide a nice review of some mechanics-related concepts and skills).

I strongly suggest that instructors explicitly and early on address the issues of "bad terminology." Two very different concepts are given the same name of **electric field**:

- The first concept is the space surrounding electric charges that possesses some specific properties (the most fundamental being that another charge placed in the vicinity of the existing charge would experience a force).
- The second concept is the ratio (F/q) where **F** is the force experienced by the probe charge and q is the magnitude of that charge.

I would recommend that, as the students study electrostatics, the term "strength of the electric field" or simply "**E**" be used rather than the conventional and misleading "electric field." Note that "the potential of the electric field" is a perfectly reasonable term because it describes the **characteristic of** what really **is** an electric field.

Questions

1. In an experiment, small positive probe charges are placed at a distance r from a large positive point charge Q.

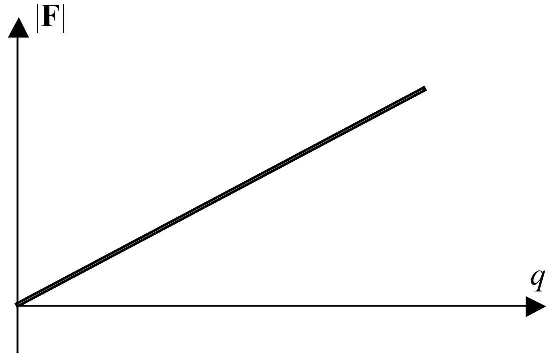
   Create the graph $|\mathbf{F}|$ vs. q where $|\mathbf{F}|$ is the magnitude of the force acting on the probe and q is the magnitude of the charge of the probe. Answer the following questions:
   a. What is the slope of the graph equal to?

b. Does the slope depend on Q? r? q?

c. What is the physical meaning of the slope?

d. Does it matter where, exactly, the probes are placed in space as long as they are placed a distance r from Q?

Solutions



a. The slope equals $\frac{kQ}{r^2}$.

b. The slope depends on Q and r but not on q.

c. The slope shows the ratio $\frac{|\mathrm{F}|}{q}$, or $|\mathbf{E}|$—the strength of the electric field at a distance r from charge Q.

d. The magnitude of the force (and, therefore, the appearance of the graph) does not depend on the placement of the probes (as long as they are all a distance r from Q); however, the direction of vectors **F** and **E** will change if the location of the probes is changed.

2. Using the graph constructed in question 1, use the same set of axes to sketch the graphs $|\mathbf{F}|$ vs. q for different situations as described below. Clearly label which graph corresponds to which situation.
   a. The probe charges are placed at a distance $2r$ from charge Q.

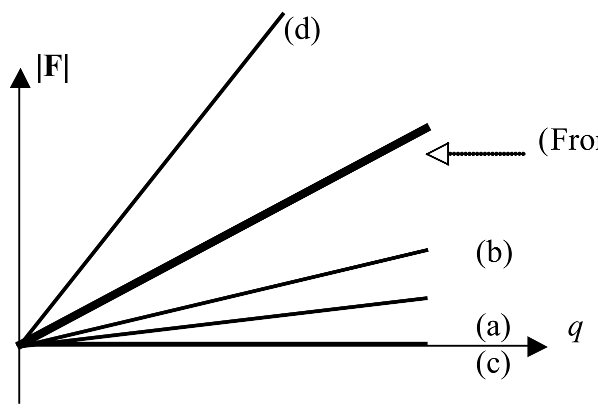
   b. Charge Q is replaced by charge $2Q$, and the probe charges are placed at a distance $2r$ from charge $2Q$.

c. Another charge Q is placed a distance $2r$ from the existing charge Q, and the probe charges are placed halfway between charges Q.

d. Charge $-Q$ is placed a distance $2r$ from the existing charge Q, and the probe charges are placed halfway between charges Q.

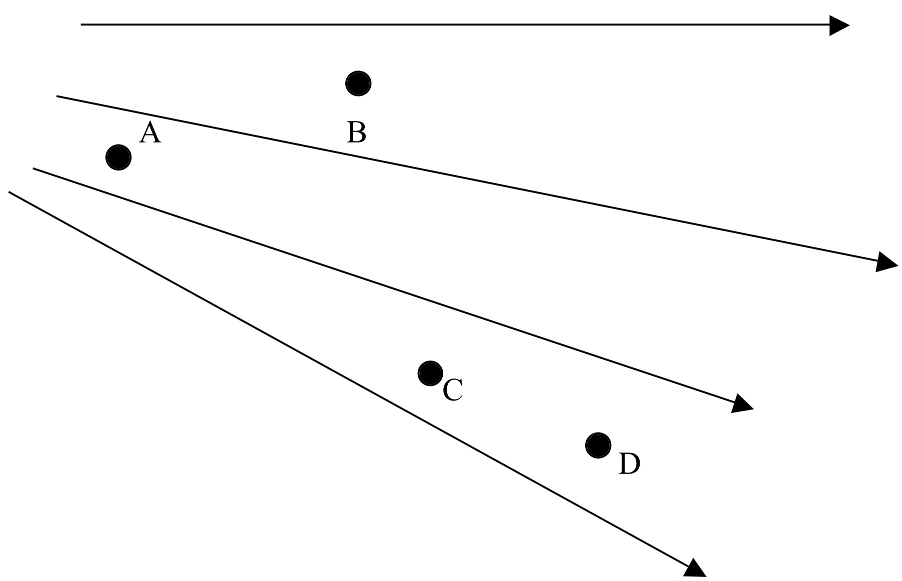
What is the meaning of the slope of these graphs?

Solutions



The slope of each graph indicates the magnitude of the net electric field at the location where the probes are placed.

3. The diagram below shows the electric field lines describing the field due to an unknown charge distribution.



The questions below refer to points A, B, C, and D, located in the field as shown in the diagram.

a. At which point would a proton experience the greatest force? The smallest force? Explain.

b. An electron is placed at one of the points and released. As the electron moves, it passes through another point. Most likely, the electron is released from point ___ and then passes through point ____.

c. Which of the points has the highest potential? The lowest potential?

d. At which of the points should one place a proton for it to have the highest potential energy? The lowest potential energy?

e. At which of the points should one place an electron for it to have the highest potential energy? The lowest potential energy?

f. Sketch the equipotential surfaces passing through each point.

Solutions

a. A, D, the lines diverge in the direction of decreasing E.

b. D, C

c. A, D

d. A, D

e. D, A

f. Curves that cross each electric field line at right angles

4. Two particles of charge $+q$ each are located as shown.

   a. For each of the points A, B, C, and D, draw the vector representing the net electric field due to both particles. Make sure that your vectors are drawn approximately to scale.

   b. Compare the potentials of the following pairs of points:

   V_A V_C

   V_C V_D

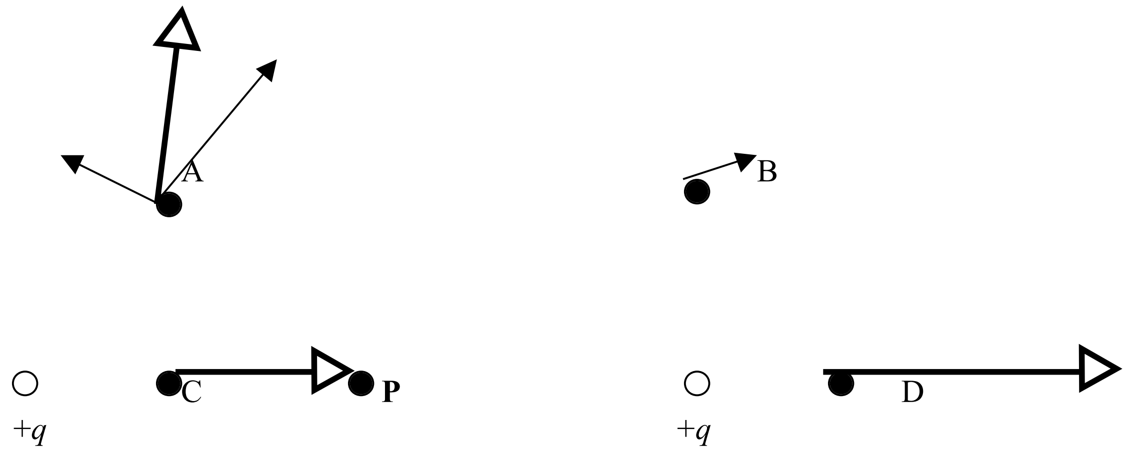
   V_B V_C

   c. Show a point on the diagram at which the electric field is zero. Label that point *P*. How does V_E compare to V_C? Explain.

   d. Repeat parts (a) and (b) if the particle on the left has the charge of $-q$ while the particle on the right still has the charge of $+q$.

Solutions

a.



b. $V_A < V_C$

$V_C > VD$

$V_B < V_C$

c. See the diagram above. Point P is halfway between the charged particles. As for potentials, if the distance between the particles is $2a$,

then $V_E = kq\left(\frac{1}{a} + \frac{1}{a}\right) = kq\left(\frac{2}{a}\right)$.

On the other hand, $V_c = kq\left(\frac{1}{a-x} + \frac{1}{a+x}\right) = kq\left(\frac{2a}{a^2 - x^2}\right)$, where x is a positive number. Comparing $\frac{2}{a}$ and $\frac{2a}{a^2 - x^2}$, one can see that the latter is greater: $V_C > V_E$.

Special Focus: Graphical Analysis

The First Law of Thermodynamics and P-V Diagrams

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A frequent question on the AP Physics B Exam deals with what is commonly referred to as a P-V diagram. An ideal gas undergoes changes of state as a result of work processes, heat processes, or both. The pressure of the gas is then plotted versus volume, and questions are asked about the state of the gas and how much work is done or heat is transferred during the different processes. As common as these questions are on the exam, they are not always a point of emphasis in the various textbooks used for the B-level course. The object of this article is to present a coherent approach to this material based on the ideal gas law and the first law of thermodynamics.

Internal Energy

Students typically learn about mechanical energy during the first semester of the B-level course. Analysis of the work necessary to accelerate an object or to reposition the object leads to the concepts of kinetic and potential energy. Should only conservative forces act on an object, the sum of the kinetic and potential energies will not change. This sum, the total mechanical energy, is associated with macroscopic motions. However, the objects we use in class to demonstrate mechanical energy (balls, textbooks, weights, springs, and so on) are all complicated systems composed of many molecules or atoms. Even when the center of mass of these objects is stationary, there is considerable energy within the object due to molecular motions and positions. I keep a small demonstration piece consisting of a dozen small spheres interconnected by springs on my front desk. Using this Einstein model of a solid, I can demonstrate kinetic and potential energy at the molecular level. The sum total of the kinetic and potential energy at the molecular level is called the **internal energy** of the system:

$$U = KE + PE \quad \text{(molecular level)}.$$

Of course, the springs in the demo piece are representative of an electric interaction between the molecules, and in a solid this interaction is strong enough to keep each molecule close to its equilibrium position. In a liquid, the molecules are still very close to each other, but the interaction is not strong enough to keep the molecules at fixed lattice sites. In a gas, the interaction is relatively weak (more precisely, the interaction energy is small compared with the average kinetic energy of a molecule), and the molecules can readily move throughout the enclosed volume if the density of the gas is not too great. In an ideal gas, the interaction energy between molecules is neglected, that is, there is no internal potential energy. This means that the internal energy of an ideal gas is the sum total of the kinetic energy of the molecules. A standard derivation in most textbooks based on kinetic molecular theory relates the average kinetic energy of a molecule to the temperature:

$$KE_{avg} = \frac{3}{2}kT.$$

In this equation, k is Boltzmann's constant, $1.38 \times 10^{-23}\ \frac{\text{J}}{\text{K}}$, and temperature is expressed in kelvins. For an ideal gas containing N molecules, the internal energy will just be N times this since there is no potential energy:

$$U = N\left(\frac{3}{2}KE_{avg}\right) = \frac{3}{2}NkT.$$

Problem statements usually give the amount of substance in moles, so you can write:

$$U = \frac{3}{2}NkT = \frac{3}{2}\left(\frac{N}{6.02 \times 10^{23}}\right)\left(6.02 \times 10^{23}k\right)T = \frac{3}{2}nRT.$$

In the last equation, R is the ideal gas constant with a value of $8.315\frac{\text{J}}{\text{mol} \cdot \text{K}}$. Actually, the formula $U = \frac{3}{2}nRT$ is only true for a monatomic ideal gas. For more complex molecules, rotational motion will also contribute significantly to the *KE*. However, this level of complexity has never been a point of emphasis on the AP Exam, so a teacher's emphasis should be on monatomic ideal gases. It is worth emphasizing here that the internal energy of an ideal gas depends only on the amount of substance and the temperature. To change the internal energy of a fixed amount of gas, the temperature must change.

Example: Calculate the internal energy of the air in a typical room with volume 40 m^3. Treat the air as if it were a monatomic ideal gas at 1 atm = 1.01×10^5 Pa.

You can use the gas law $PV = nRT$ to express the internal energy in terms of pressure and volume:

$$U = \frac{3}{2}nRT = \frac{3}{2}PV = \frac{3}{2}\left(1.01 \times 10^5\right)\left(40\right) = 6.06 \times 10^6\ \text{J}.$$

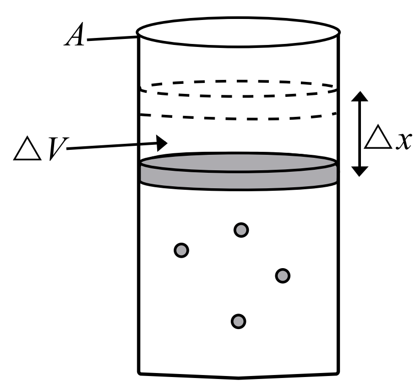
Special Focus: Graphical Analysis

Energy Transfers: Work and Heat

The first law deals with changes in the internal energy of a system. In teaching the first law, it is important to emphasize that changes in the internal energy of a system can only occur if the system is not isolated. This means that the system is embedded in its surroundings in a way that there can be an energy transfer. There are two types of energy transfer, and the difference between the two is determined not by what occurs in the system but by what occurs in the surroundings.

Work is an energy transfer between a system and its surroundings that is a result of organized motion in the surroundings. You can increase the internal energy of a wood block by rubbing it vigorously. You can increase the internal energy of a glass of water by stirring it rapidly. You can decrease the internal energy of a gas by letting it expand against some external pressure applied by a piston. In each example, the surroundings (rubbing cloth, stirrer, piston) are systems consisting of many particles that are all undergoing an organized motion as a whole to facilitate the energy transfer.

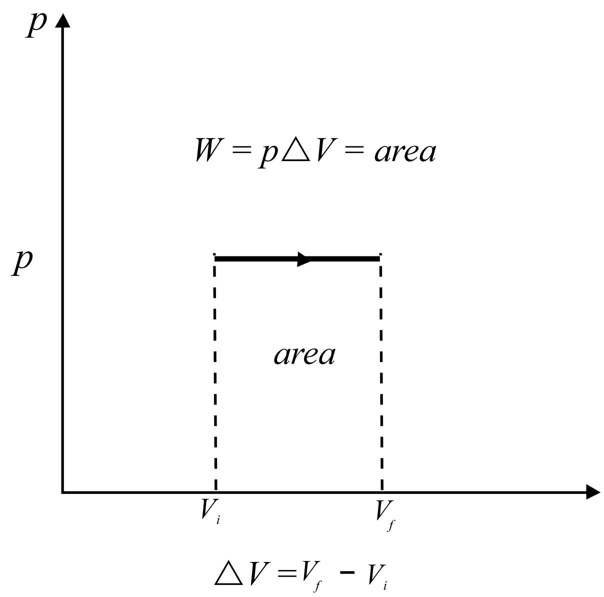
The system consisting of an ideal gas enclosed in a container with a moveable piston is commonly used to illustrate many of the concepts of thermodynamics. Let's look at it in more detail. Suppose the gas is allowed to expand slowly in such a way that at any point in the process the pressure is well defined and the system can be thought of as being close to equilibrium every step of the way. Such a process is said to be quasi-static. Since the gas is always in equilibrium, the gas pressure is always equal to the external pressure exerted by the piston, and the work done by the gas on the piston is just the negative of the work done on the gas by the piston. Let us further assume that while the gas expands, the pressure P remains constant, an **isobaric** process. This could be achieved by immersing the gas container in consecutively warmer water baths. After a time, the piston with cross-sectional area A would have expanded a distance Δx.



Since $P = \frac{F}{A}$, the force that the gas exerts on the piston is given by $F = PA$. Then the work done by the gas during the expansion is given by:

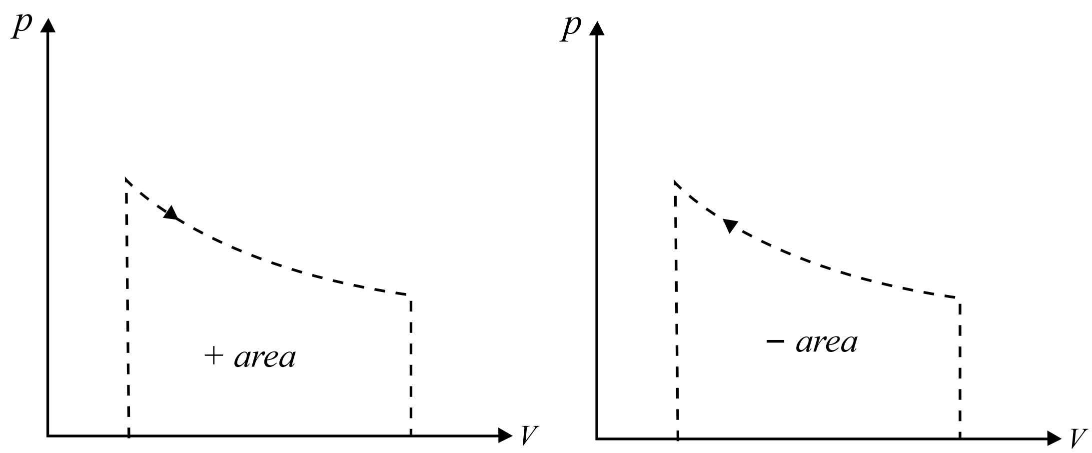
$$W_{by} = F\Delta x = PA\Delta x = P\Delta V.$$

It is a simple matter to display this process on a graph that plots pressure versus volume with an arrow to show the sequence of the process.



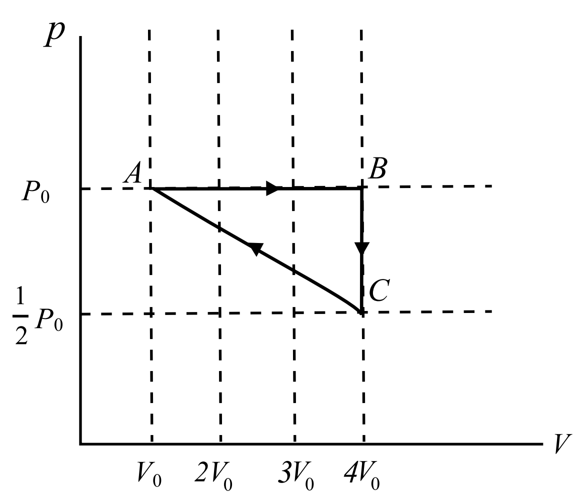
Note that the work done by the gas is just the area under the "curve," i.e., between the graph and the V-axis. By convention, this area is positive for positive work done *by* the gas, as in this example. For our quasi-static process, this would correspond to negative work done *on* the gas. The distinction between work done on the gas and work done by the gas is one that is often made on the AP Exam, so a teacher should make this distinction clear at this point. As long as the process is quasi-static so that the pressure is well defined at each step of the way, the area under the P-V curve will always be the work done by the gas during the process. This follows from the fact that any curve can be approximated by a series of steps over which the pressure is constant. In the limit of infinitesimal steps, you get the exact answer, the sum of all the infinitesimal areas, the area under the curve.

Special Focus: Graphical Analysis



A special case is that of a **cycle** in which the system is brought back to its original state after going through several processes.

> **Example:** One mole of monatomic ideal gas is enclosed under a frictionless piston. A series of processes occur, and eventually the state of the gas returns to its initial state with a *P*-*V* diagram as shown below. Answer the following in terms of P_0, V_0, and R.



a. Find the temperature at each vertex.
b. Find the change in internal energy for each process.
c. Find the work by the gas done for each process.

a. Use the gas law to find the temperatures at A, B, C:

$$P_0V_0 = nRT_A \Rightarrow T_A = \frac{P_0V_0}{R}.$$

Similarly, $T_B = \dfrac{4P_0V_0}{R}$ and $T_C = \dfrac{2P_0V_0}{R}$.

b. Since the internal energy depends only on temperature, the change in internal energy for each process depends only on the temperature difference that occurs during the process:

$$\Delta U = \frac{3}{2}nR\Delta T \Rightarrow \Delta U_{A\to B} = \frac{3}{2}R\left(\frac{4P_0V_0}{R} - \frac{P_0V_0}{R}\right) = \frac{9}{2}P_0V_0.$$

Similarly,

$$\Delta U_{B\to C} = \frac{3}{2}R\left(\frac{2P_0V_0}{R} - \frac{4P_0V_0}{R}\right) = -3P_0V_0,$$

$$\Delta U_{C\to A} = \frac{3}{2}R\left(\frac{P_0V_0}{R} - \frac{2P_0V_0}{R}\right) = -\frac{3}{2}P_0V_0.$$

c. To find the work done by the gas, find the area under each segment, remembering the sign convention.

$$W_{by}^{A\to B} = 3P_0V_0$$
$$W_{by}^{B\to C} = 0$$
$$W_{by}^{C\to A} = \text{triangle} + \text{rectangle}$$
$$= -\frac{1}{2}(3V_0)\left(\frac{1}{2}P_0\right) - \left(\frac{1}{2}P_0\right)(3V_0) = -\frac{9}{4}P_0V_0.$$

There are two things to note about part (c) that are true in general:

1. For a constant volume process like B→C, no work is done by the gas.

2. The total work done for the entire cycle is the area enclosed within the graph.

   In this example, the sum of the work is $W_{total} = 3P_0V_0 - \frac{9}{4}P_0V_0 = \frac{3}{4}P_0V_0$, the same as the area of the enclosed triangle.

Heat is an energy transfer between a system and its surroundings that is the result of random motion in the surroundings. Note the difference between a work process and a heat process. In the former, there must be organized motion in the surroundings, but in the latter, the energy transfer is a result of random motion in the surroundings. When a glass of water is placed on a hot plate, energy will spontaneously leave the hot plate and cause the internal energy of the water to increase. Heat will always flow spontaneously from the system at higher temperature to the system at lower temperature, but heat can be made to flow in the opposite direction as well if work is done in the process. For example, a refrigerator relies on work done by its compressor to move heat from a cold freezer into a warm kitchen.

Heat is a term that is used all the time in everyday life. In this context, it's okay to talk about the "amount of heat in a hot object," but in the classroom it is important that a teacher emphasize the precise meaning of this term. A hot object may contain a lot of internal energy, but it does not contain heat. Unfortunately, physics textbooks are often guilty of sloppy usage. Phrases like "as friction slowed the block, heat was generated in the sliding surface" are not difficult to find. As the block slows, the organized motion of the block does work on the sliding surface and increases its internal energy. Such sloppy usage masks the essential difference between a work process and a heat process, and a solid grasp of the first law of thermodynamics cannot be achieved without understanding this distinction.

The First Law of Thermodynamics

Heat processes and work processes account for all possible energy transfers to a system. It therefore follows from conservation of energy that the total change in the internal energy of a system is the sum of the work done on the system and the heat transferred to the system. This is the first law of thermodynamics:

$$\Delta U = W_{on} + Q_{\text{into}}.$$

This equation is often written simply as $\Delta U = W + Q$, but you have to keep the sign conventions in mind. It is probably easiest to remember the conventions for W and Q by imagining processes where only one of them is present. If $Q = 0$, then ΔU is positive if positive work is done *on* the system. If $W = 0$, then ΔU is positive if heat flows *into* the system.

Let's go back to the *P*-*V* diagram example and calculate the heat transfer for each process. Since the work done by the gas and the change in the internal energy have already been calculated, it is a simple matter to calculate the heat transfer from the first law, but be careful of signs. Remember that for our quasi-static process, $W_{by} = -W_{on}$.

$$\Delta U_{A\to B} = W_{on}^{A\to B} + Q_{\text{into}}^{A\to B} \Rightarrow Q_{\text{into}}^{A\to B} = \Delta U_{A\to B} + W_{by}^{A\to B},$$
$$Q_{\text{into}}^{A\to B} = \frac{9}{2}P_0V_0 + 3P_0V_0 = \frac{15}{2}P_0V_0.$$

$$\Delta U_{B\to C} = W_{on}^{B\to C} + Q_{\text{into}}^{B\to C} \Rightarrow Q_{\text{into}}^{B\to C} = \Delta U_{B\to C} + W_{by}^{B\to C},$$
$$Q_{\text{into}}^{B\to C} = -3P_0V_0 + 0 = -3P_0V_0.$$

$$\Delta U_{C\to A} = W_{on}^{C\to A} + Q_{\text{into}}^{C\to A} \Rightarrow Q_{\text{into}}^{C\to A} = \Delta U_{C\to A} + W_{by}^{C\to A},$$
$$Q_{\text{into}}^{C\to A} = -\frac{3}{2}P_0V_0 - \frac{9}{4}P_0V_0 = -\frac{15}{4}P_0V_0.$$

The efficiency of a **cycle** is defined as the ratio of the work done by the gas to the heat Q_{in} that flows *into* the system. Any heat that is expelled into the surroundings is not included in the calculation of Q_{in}. From the point of view of efficiency, this expelled heat is lost, and its energy is not used by the system:

$$e = \frac{W}{Q_{in}}.$$

In the example above, you get:

$$e = \frac{\left(\frac{3}{4}P_0V_0\right)}{\left(\frac{15}{2}P_0V_0\right)} = 0.1.$$

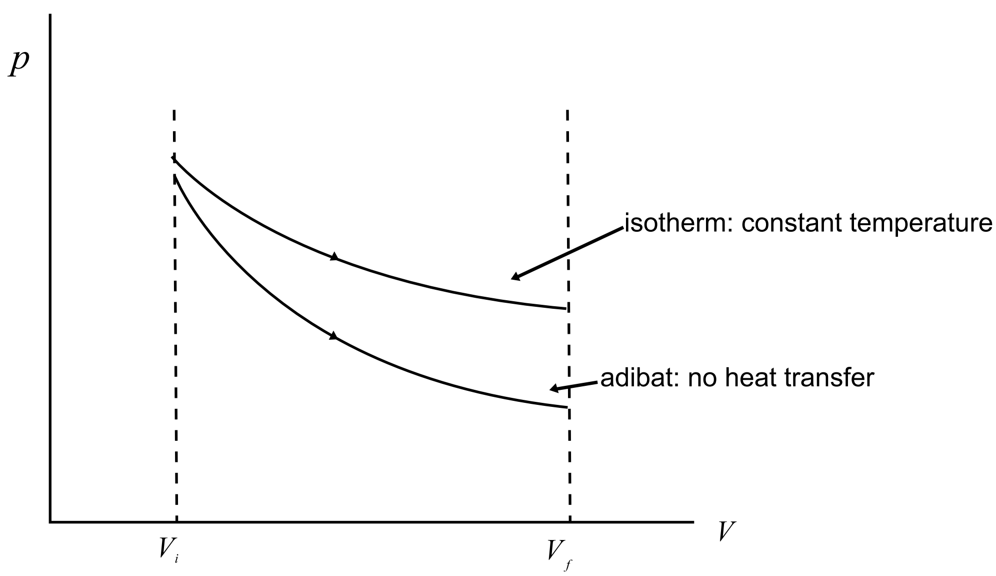
Besides the constant pressure (isobaric) and constant volume processes involved in this example, there are two other processes the student of AP Physics must be familiar with. An **isothermal** process is one that occurs at constant temperature. The gas-piston container in our example could expand isothermally if it were kept immersed in a large hot-water bath while the gas expanded. Since the temperature doesn't change during an isothermal process, there is no change in internal energy. The first law then tells you that the work done by the gas is just equal to the heat that flows into the system from the bath:

$$\Delta U = 0 = W_{on} + Q_{\text{into}} = -W_{by} + Q_{\text{into}} \Rightarrow W_{by} = Q_{\text{into}} \text{ (isothermal process).}$$

An **adiabatic** process is one that occurs without the exchange of heat with the surroundings. If the gas-piston system were insulated so that heat could not get in or out, any expansion or compression would occur adiabatically. Since $Q = 0$ for an adiabatic process, the first law tells you that the change in internal energy is just equal to the work done on the system:

$$\Delta U = W_{on} + 0 = -W_{by} \text{ (adiabatic process).}$$

When a gas expands adiabatically, the work done in the expansion comes at the expense of the internal energy of the gas, causing the temperature of the gas to drop. The figure below shows *P*-*V* diagrams for these two processes.



The figure compares two processes that begin with the same state and involve expansion to the same final volume. For the isothermal process, the product of $P \cdot V$ remains constant since T remains constant. Since the temperature must decrease for the adiabatic process, it follows that the final pressure must be less for this process. Thus the adiabat lies below the isotherm.

Let's look at one more example that incorporates many of the AP points of emphasis.

Example: One mole of ideal gas is at pressure P_0 and volume V_0. The gas then undergoes three processes:

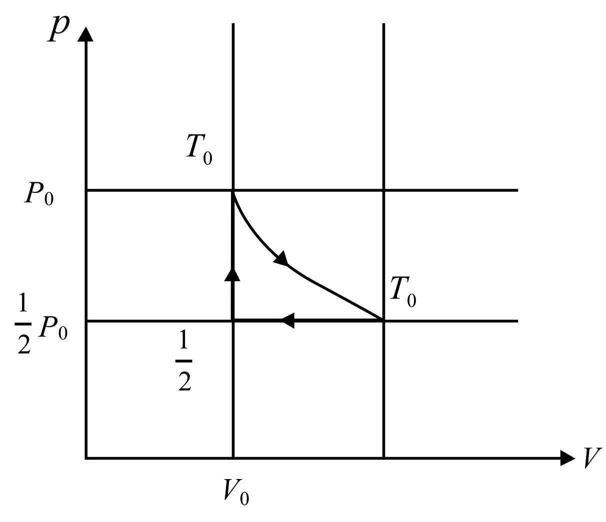
1. The gas expands isothermally to $2V_0$ while heat Q flows into the gas.
2. The gas is compressed at constant pressure back to the original volume.
3. The pressure is increased while holding the volume constant until the gas returns to its initial state.

a. Draw a *P*-*V* diagram that depicts this cycle. Label relevant points on the axes. In terms of T_0, the initial temperature, label each vertex with the temperature of the gas at that point.

For the remaining sections, answer in terms of T_0, *Q*, and *R*.

b. Find the change in internal energy for each leg of the cycle.
c. Find the work done by the gas on each leg of the cycle.
d. Find the heat that flows into the gas on legs 2 and 3.
e. Find the efficiency of this cycle.

a.



The two ends of the isotherm will both be at T_0. Since the product of $P \cdot V$ is constant along an isotherm, an expansion to twice the volume implies a pressure reduction to half the original pressure. Applying the gas law to the lower-left vertex then yields:

$$\left(\frac{1}{2}P_0\right)V_0 = nRT \Rightarrow T = \frac{1}{2}T_0 .$$

b. 1. $\Delta U = 0$ (isotherm).

2. $\Delta U = \frac{3}{2}R\left(\frac{1}{2}T_0 - T_0\right) = -\frac{3}{4}RT_0 .$

3. $\Delta U = \frac{3}{2}R\left(T_0 - \frac{1}{2}T_0\right) = +\frac{3}{4}RT_0 .$

c. 1. Since $\Delta U = 0$, it follows from the first law that $W_{by} = Q$.

2. $W_{by} = area = -\frac{1}{2}P_0V_0 = -\frac{1}{2}RT_0 .$

3. $W_{by} = 0$ (constant volume process).

d. Use the first law since both ΔU and W are known for each process.

1. $\Delta U = Q_{\text{into}} + W_{on} \Rightarrow Q_{\text{into}} = \Delta U + W_{by} .$

2. $Q_{\text{into}} = -\frac{3}{4}RT_0 - \frac{1}{2}RT_0 = -\frac{5}{4}RT_0 .$

3. $Q_{\text{into}} = +\frac{3}{4}RT_0 + 0 = \frac{3}{4}RT_0 .$

e. Heat flows into the system on both legs 1 and 3. Use the definition of efficiency:

$$e = \frac{W}{Q_{in}} = \frac{Q - \frac{1}{2}RT_0}{Q + \frac{3}{4}RT_0} .$$

Conclusion

Older versions of the AP Physics B Exam often gave C_p and C_V, the specific heats at constant pressure and volume, within the problem statement involving a *P*-*V* diagram. In fact, this information was never actually necessary to solve the problem. A solid grasp of the first law of thermodynamics and facility with the ideal gas law are the only tools a student needs to deal with these problems.

Graphing Analysis in Modern Physics

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The topics covered in this paper are energy-level diagrams and graphs of the photoelectric effect. Background information on the topics can be found in all major textbooks. However, a brief summary is included at the beginning of each topic followed by exercises that involve the use of diagrams or graphs.

Part 1: Energy-Level Diagrams

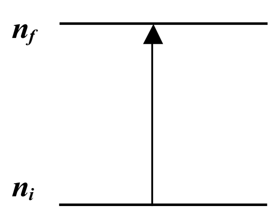
Radiation is absorbed or released when an electron changes from one stationary state to another. The energy of the emitted or absorbed photon is said to be quantized and is equal to the difference in the energy between these two states:

$$\Delta E = E_f - E_i$$

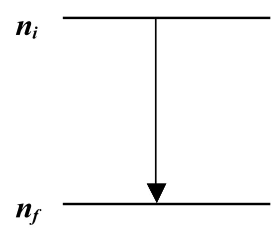
The sign of the energy change depends on whether the photon is absorbed or emitted. For example, when an electron in a hydrogen atom changes from $n = 2$ to $n = 1$, light is emitted and the energy change is negative, since energy is lost by the atom. If the transition is from $n = 1$ to $n = 2$, energy is absorbed and the sign of the energy change is positive.

The transitions on an energy-level diagram are shown with arrows.

The following diagram shows the absorption of a photon to promote the electron to a higher energy level.



The next diagram shows the emission of a photon when the electron falls from a higher to a lower energy level.



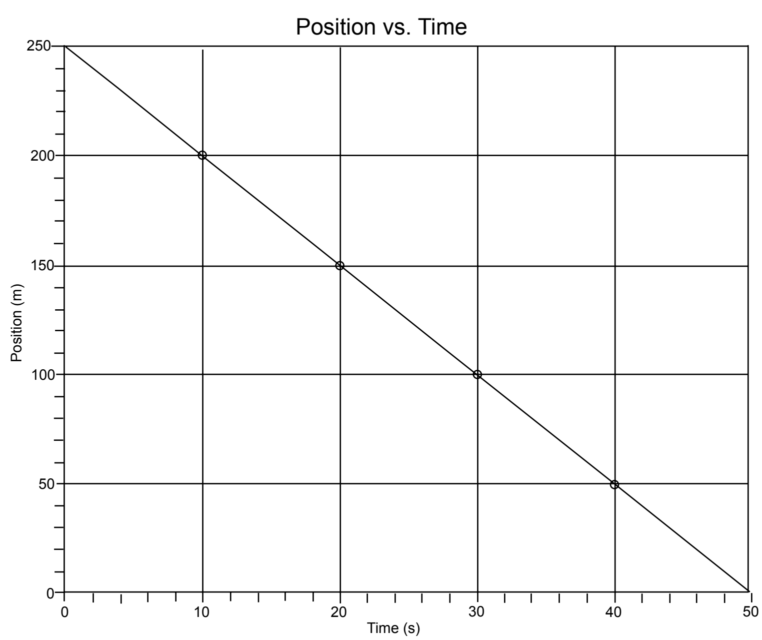
Graphing Techniques

Graphs are valuable tools in physics since they portray the relationship between two variables and can be used to verify if experimental values follow a theoretical relationship.

The three most common types of graphs encountered in physics are a straight line, a parabola, and a hyperbola. Some of the tools used in analysis are related to finding the equations that correspond to the slopes of a particular curve. A technique that simplifies finding slopes is to "linearize" the curve, that is, to make the curve into a straight line. The following examples show the three types of graphs and the "linearization" of parabolas and hyperbolas.

1. Straight Line
Equation: $y = mx + b$.

The following graph shows the motion of a cyclist moving south at constant velocity.



From the graph we learn that the cyclist started at a point 250 m north from the final location. The straight line indicates that the cyclist's displacement is directly proportional to the time.

To find the velocity, we calculate the slope of the line:

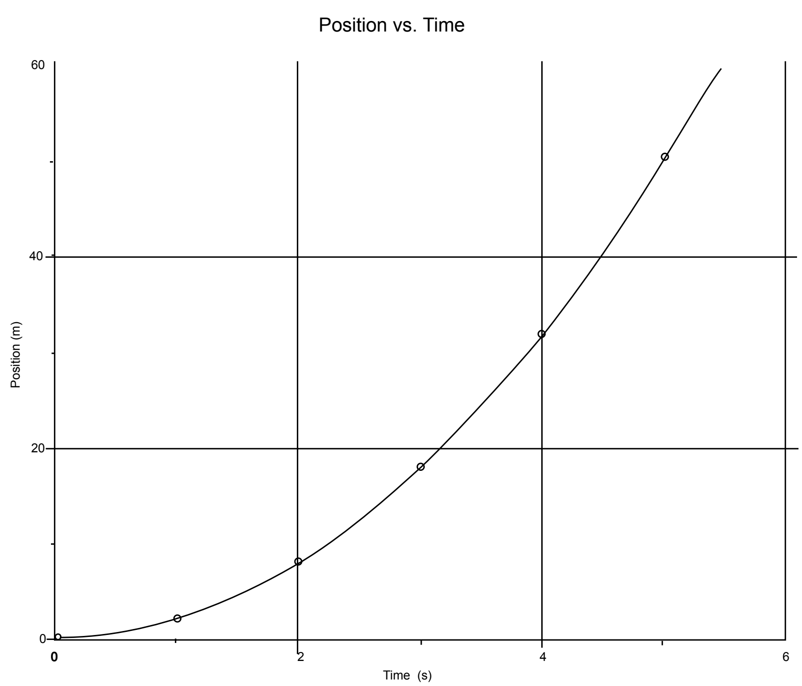
$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{50\text{ m} - 200\text{ m}}{40\text{ s} - 10\text{ s}} = -5\text{ m/s}.$$

The equation of the line becomes: $d = -5(t) + 250$.

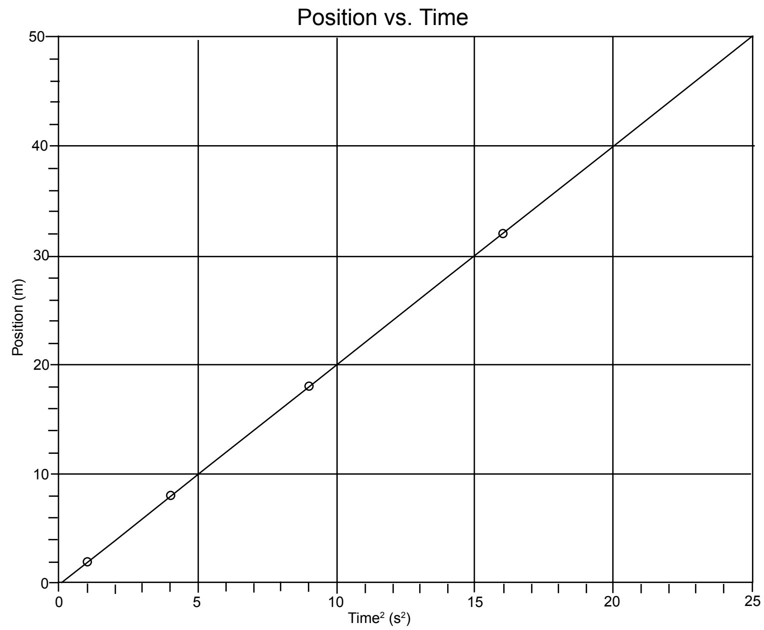
2. Parabola

Equation: $y = mx^2$.

The graph below depicts the velocity of a steel ball that rolls down an incline.



From the graph we can see that the displacement of the steel ball varies directly with the square of the time. Therefore, to linearize the graph, we have to plot d vs. t^2.



The acceleration of the ball can be found by calculating the slope of the line:

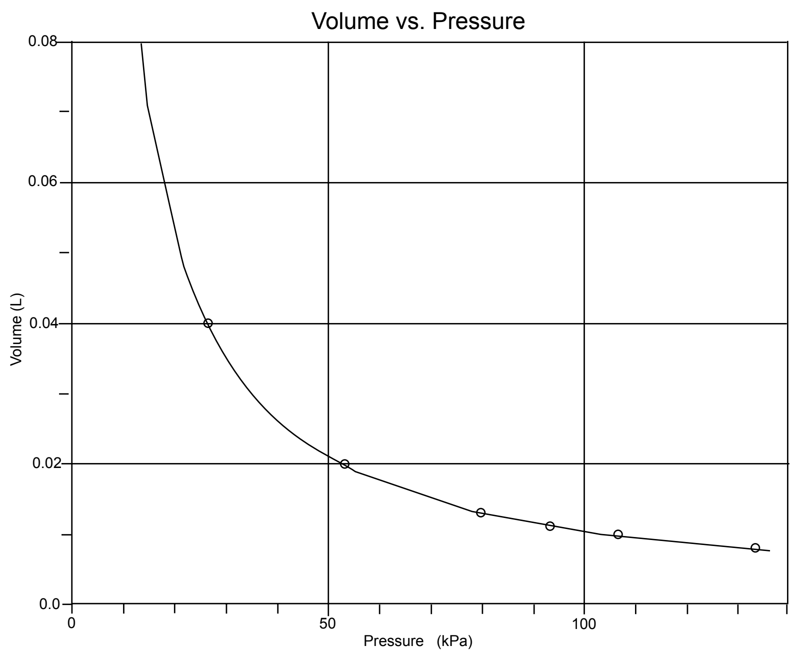
$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{50\text{ m} - 8\text{ m}}{25\text{ s}^2 - 4\text{ s}^2} = 2\text{ m/s}^2.$$

The equation of the curve becomes: $d = 2t^2$.

3. Hyperbola

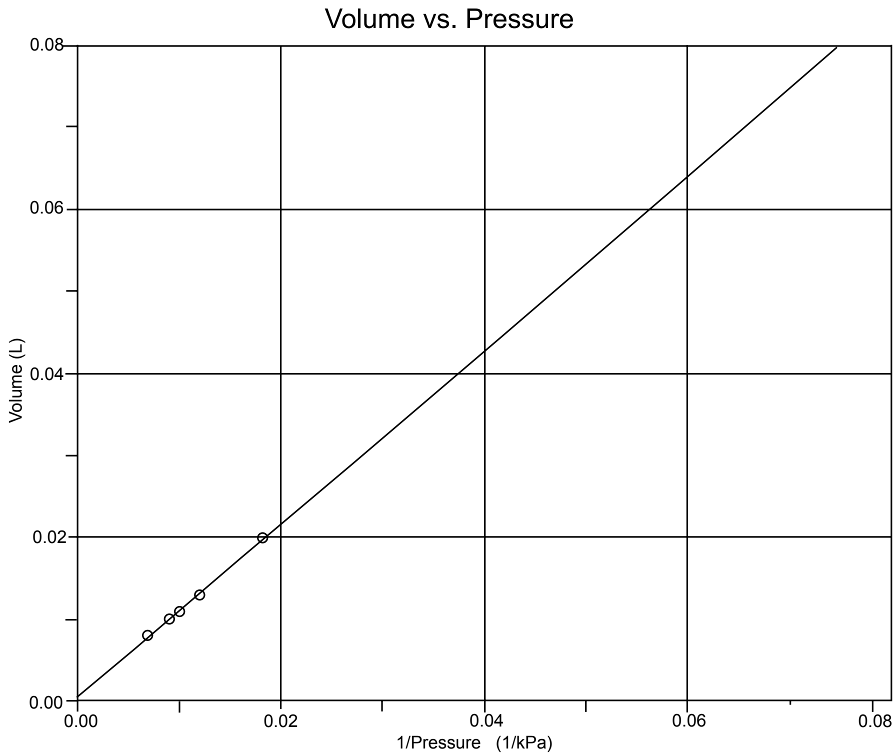
Equation: $y = \frac{m}{x}$.

The following graph shows data from an experiment designed to verify Boyle's law.



The graph shows an inverse relationship between the pressure and the volume.

To linearize the graph, we plot V vs. $\frac{1}{P}$ to obtain a straight line with a constant slope that proves that PV = constant.



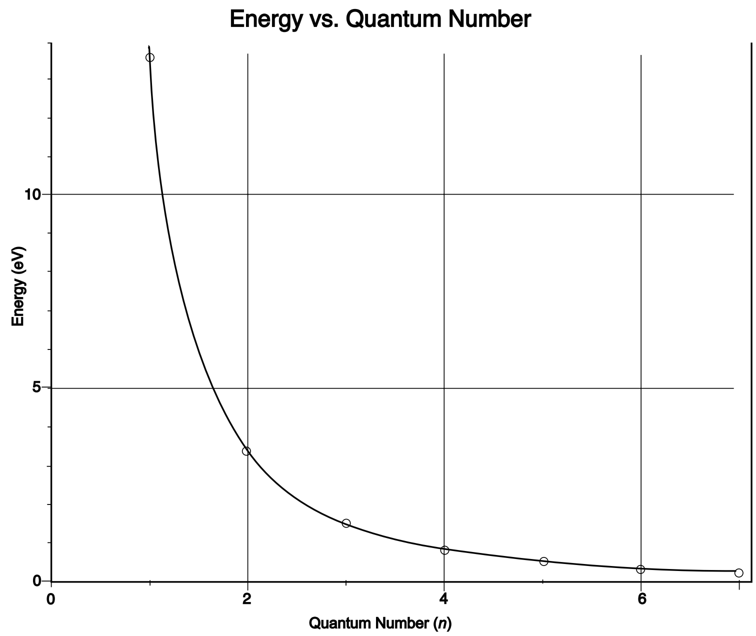
Exercise 1

The data from an experiment of the spectral lines of the hydrogen atom are given in the table below. Apply "linearizing" techniques to derive an equation in terms of the energy and the quantum numbers for the hydrogen atom.

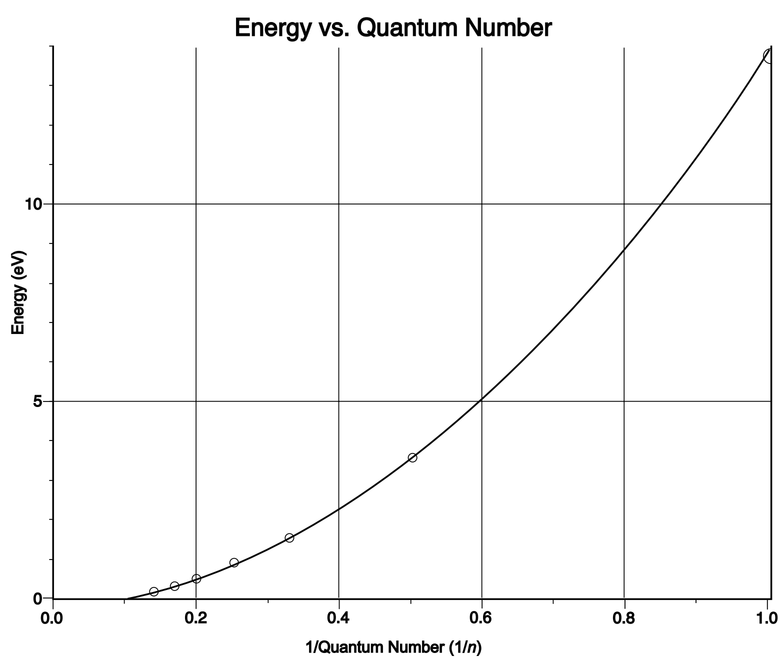
Quantum Number (n)	|Energy| (eV)
1	13.6
2	3.4
3	1.5
4	0.8
5	0.5
6	0.3
7	0.2

Solution

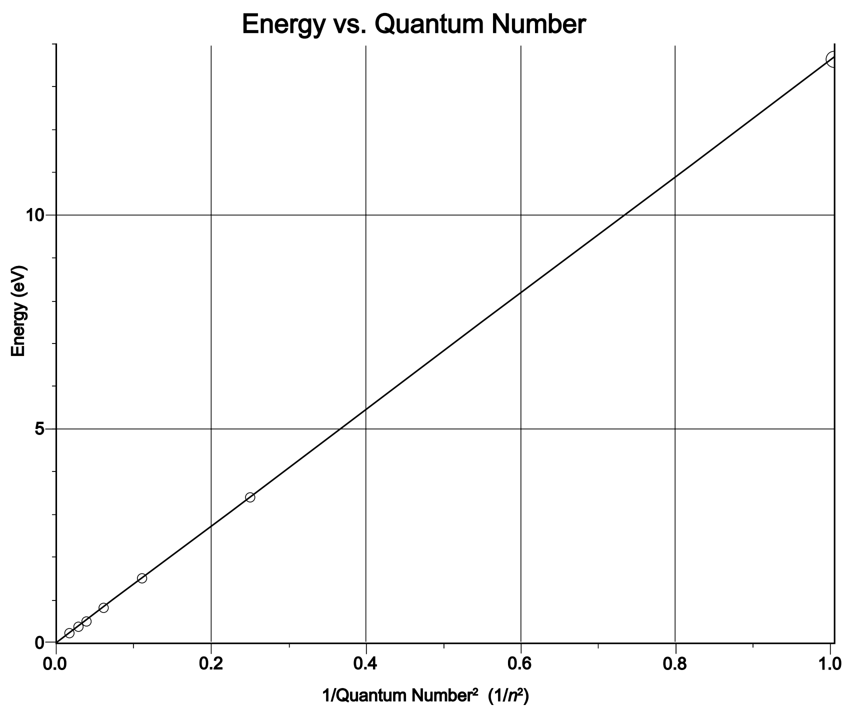
The graph below is obtained.



The graph suggests that the energy varies inversely with the quantum number; therefore we need to linearize the graph using E vs. $\frac{1}{n}$ to obtain the following graph.



From the graph we see that the energy varies with the inverse of the square of the quantum number. Doing a second linearization, we see that the graph of E vs. $\frac{1}{n^2}$ yields a straight line.



We can now use the graph to find the slope:

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{13.6\text{ eV} - 0.8\text{ eV}}{1 - 0.06} = 13.6\text{ eV}.$$

The equation of the line is: $y = mx + b$.

By substituting, we obtain: $\left|E\right| = 13.6\text{ eV}\left(\frac{1}{n^2}\right) + 0$.

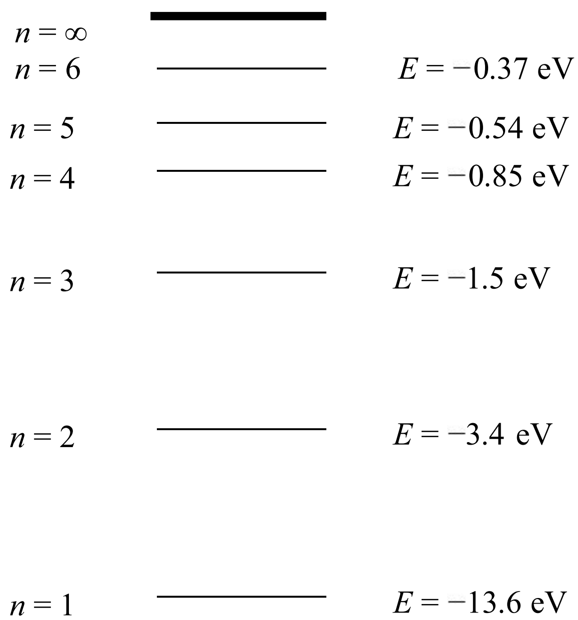
The final answer is:

$$\left|E\right| = \frac{13.6}{n^2}.$$

Exercise 2

The **Balmer series** is the series of transitions and resulting emission lines of the hydrogen atom characterized by the electron transitioning from $n \geq 3$ to $n = 2$.

On the energy-level diagram below, use arrows to represent the energy-level transitions of the Balmer series that correspond to the visible line spectrum of hydrogen.



Solution

For the Balmer series:

$$E_2 - E_6 = -3.4 \text{ eV} - (-0.37 \text{ eV}) = 3.03 \text{ eV},$$

$$E_2 - E_5 = -3.4 \text{ eV} - (-0.54 \text{ eV}) = 2.86 \text{ eV},$$

$$E_2 - E_4 = -3.4 \text{ eV} - (-0.85 \text{ eV}) = 2.55 \text{ eV},$$

$$E_2 - E_3 = -3.4 \text{ eV} - (-1.5 \text{ eV}) = 1.9 \text{ eV}.$$

The energy is given by: $E = \frac{hc}{\lambda}$.

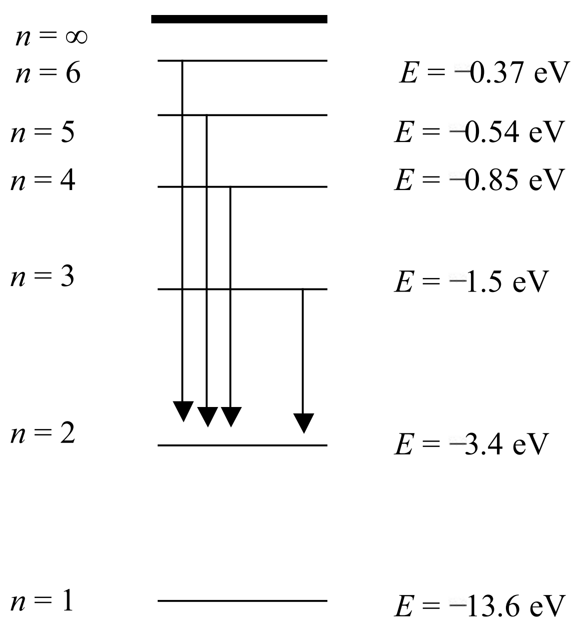
Solving for the wavelength: $\lambda = \frac{hc}{E}$,

where hc = 1,240 eV·nm.

By substituting in the equation, the wavelengths obtained are all in the visible part of the spectrum as follows:

410.5 nm (violet), 434.4 nm (blue), 486.5 nm (green-blue), and 656.8 nm (red).

The energy transitions of the Balmer series are:



Exercise 3

A hypothetical atom has four energy levels: 0 eV, 1 eV, 3 eV, and 6 eV.

a. Draw an energy-level diagram for this atom indicating the quantum numbers and the energies associated with them.

b. Use arrows to show all of the possible transitions between energy levels.

c. For which transition is the associated photon energy highest?

d. For which transition is the associated photon energy lowest?

e. For which transition is the associated photon wavelength longest?

f. For which transition is the associated photon wavelength shortest?

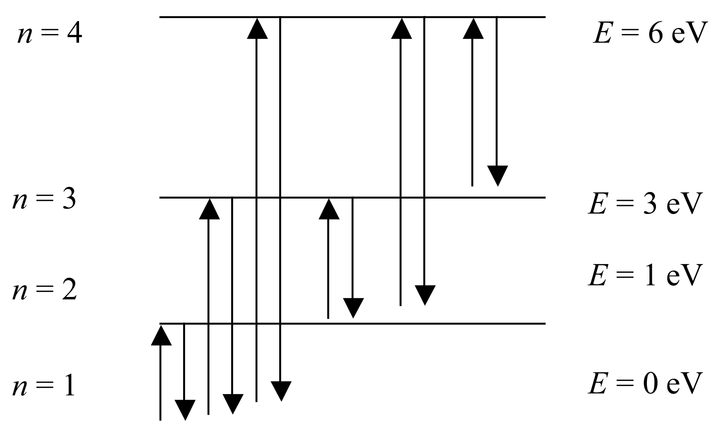
g. A photon incident on the hypothetical atom causes the electron to make a transition from the $n = 2$ orbital to the $n = 4$ orbital. What is the wavelength of the photon?

h. How many wavelengths of emitted radiation are possible when the electron returns to the $n = 2$ state?

i. An electron moving with a speed of 1.25×10^6 m/s collides with the hypothetical atom. Is the energy provided by the electron enough to excite the atom to the $n = 3$ state? Is it enough for the atom to reach the $n = 4$ state?

Solution

a and b. The 12 possible transitions are indicated by the arrows showing either absorption or emission of a photon.



c. The highest possible photon energy is 6 eV, corresponding to a transition between the $n = 1$ and $n = 4$ levels.

d. The lowest photon energy is 1 eV, corresponding to a transition between the $n = 1$ and $n = 2$ levels.

e. The longest wavelength corresponds to the lowest energy since $\lambda = \frac{hc}{E}$.

The transition between $n = 1$ and $n = 2$ corresponds to the longest wavelength.

f. The transition between $n = 1$ and $n = 4$ (highest energy) corresponds to the shortest wavelength.

g. $E_4 - E_2 = 6 \text{ eV} - 1 \text{ eV} = 5 \text{ eV}$.

$$\lambda = \frac{hc}{E} = \frac{1{,}240 \text{ eV} \cdot \text{nm}}{5 \text{ eV}} = 248 \text{ nm}.$$

h. The electron falling from the excited $n = 4$ state to the $n = 2$ state can take two paths: a direct $4 \rightarrow 2$ or through $4 \rightarrow 3 \rightarrow 2$. This would correspond to three distinct wavelengths of the emission spectra.

i. First we need to calculate the energy of the moving electron:

$$KE_e = \frac{1}{2}mv^2 = \frac{1}{2}(9.11 \times 10^{-31} \text{kg})(1.25 \times 10^6 \text{m/s})^2 = 7.12 \times 10^{-19} \text{J}.$$

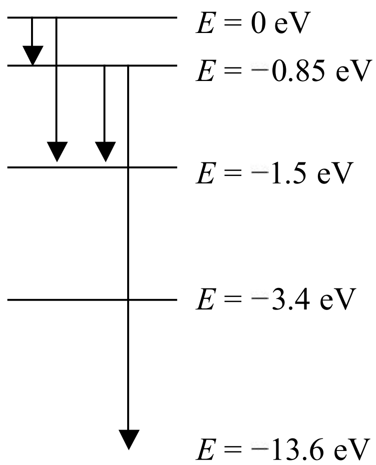
Changing the units from joules to eV:

$$KE_e = \frac{7.12 \times 10^{-19} \text{J}}{1.6 \times 10^{-19} \text{J/eV}} = 4.45 \text{ eV}.$$

The electron has enough energy to excite the atom to the $n = 3$ state but not to $n = 4$, since that will require 6 eV.

Exercise 4

The diagram shows five energy levels of the hydrogen atom. Which energy level(s) will give rise to a spectral line in the ultraviolet part of the spectrum?



Solution

The only transition possible is:

$$E_{13.6} - E_{0.85} = -13.6 \text{ eV} - (-0.85 \text{ eV}) = 12.75 \text{ eV}.$$

Exercise 5

Common fluorescent lights use an electric discharge in mercury vapor to cause atomic emissions from mercury atoms.

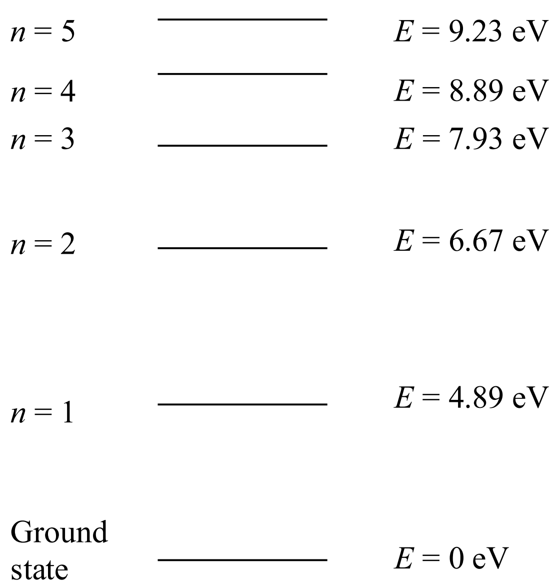
Suppose that a mercury atom is in the excited state when its energy level is 6.67 eV above the ground state. A photon of energy 2.22 eV strikes the mercury atom and is absorbed by it.

a. To what energy level is the mercury atom raised?

b. Use arrows to show the transition on the diagram.

c. List the specific transitions that produce visible light.

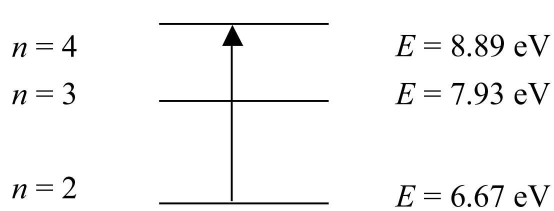
Below is a simplified energy-level diagram for the mercury atom.



Solution

a and b. $E = 6.67$ eV + 2.22 eV = 8.89 eV.

This value corresponds to the n = 4 level. The transition (absorption) is shown below.



c. The visible light range is from 400 to 700 nanometers. The range corresponds to these energies:

$$E = \frac{hc}{\lambda} = \frac{1{,}240 \text{ eV} \cdot \text{nm}}{400 \text{ nm}} = 3.1 \text{ eV}, \text{ and}$$

$$E = \frac{hc}{\lambda} = \frac{1{,}240 \text{ eV} \cdot \text{nm}}{700 \text{ nm}} = 1.77 \text{ eV}.$$

The transitions that fall within this range are 5 → 2 (2.56 eV), 4 → 2 (2.22 eV), 3 → 1 (3.04 eV), and 2 → 1 (1.78 eV).

Part 2: The Photoelectric Effect

The emission of electrons from certain metals as a result of light falling on them is called the photoelectric effect.

The main characteristics of photoelectric emission are:

1. Electrons are emitted only when the frequency of the light is above some threshold value (f_o), no matter how intense the light.

2. The maximum kinetic energy of the emitted electrons depends on the frequency of the light.

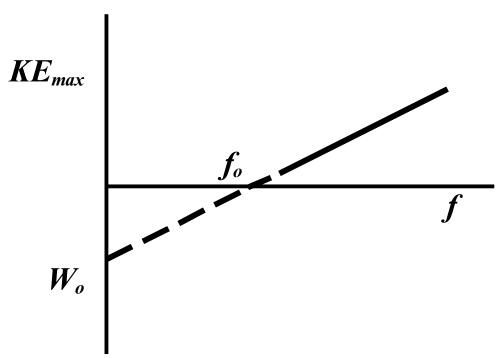
3. The photoelectrons are emitted almost at once when the light strikes.

4. The minimum energy required to release an electron from a certain metal is called the work function of the metal W_o.

5. The number of photoelectrons emitted is directly proportional to the intensity of the light.

The number of photons emitted per second = $\frac{P}{hf}$, where P is the power of the source.

A graph of the maximum kinetic energy of the emitted electrons versus the frequency of the light can be used to find the following:



Slope of the line = Planck's constant:
x-intercept: Threshold frequency
y-intercept: Work function

Special Focus: Graphical Analysis

Exercise 1

A photoelectric experiment using a mercury vapor light source was designed to investigate the following relationships:

- Current versus the intensity of light
- Stopping voltage versus the intensity of the light
- Stopping voltage versus the frequency of light using various filters

The data tables are shown below. Your task is to construct qualitative graphs to determine the relationship between each of the variables.

Table 1: Current vs. Intensity

Luminous Flux (lm)	Current (mA)
111	2.0×10^{-5}
250	4.5×10^{-5}
1,000	1.8×10^{-4}

Table 2: Stopping Voltage vs. Intensity

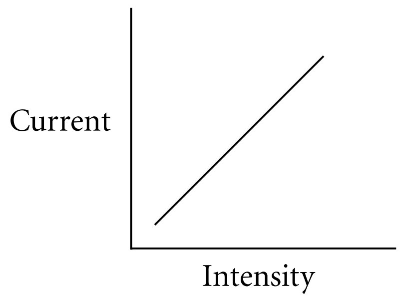
Luminous Flux (lm)	Stopping Voltage (V)
111	0.41
250	0.40
1,000	0.42

Table 3: Stopping Voltage vs. Frequency

Filter Color	Frequency (Hz)	Stopping Voltage (V)
Violet	7.4×10^{14}	0.94
Blue	6.9×10^{14}	0.71
Green	5.5×10^{14}	0.46
Yellow	5.2×10^{14}	0.40

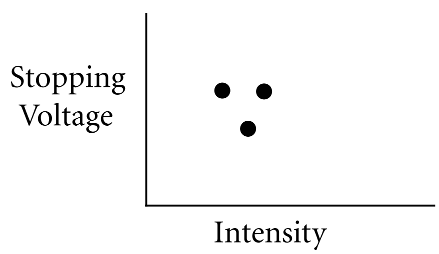
Solution

1.



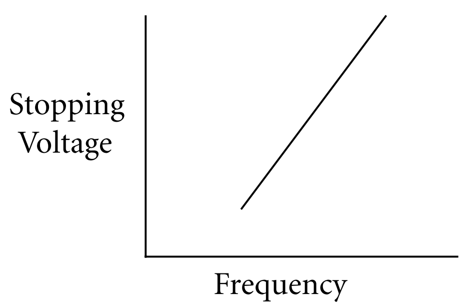
The current is directly proportional to the intensity.

2.



The intensity did not have a noticeable effect on the stopping voltage.

3.



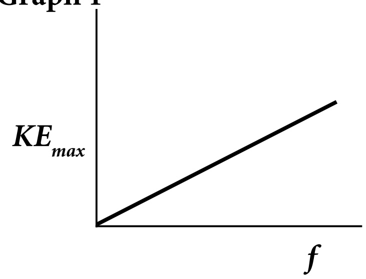
The stopping voltage is directly proportional to the frequency.

Special Focus: Graphical Analysis

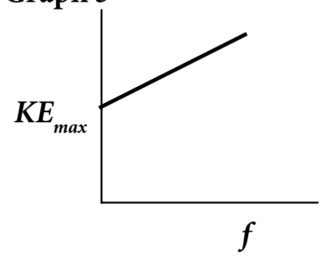
Exercise 2

Which of the graphs below shows the relationship of the kinetic energy of the ejected electrons and the frequency of the incident light?

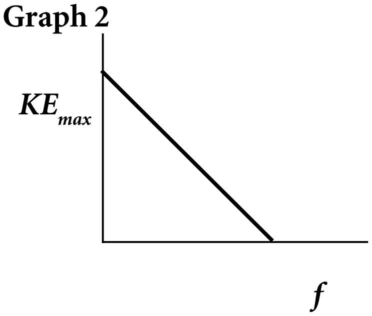
Graph 1



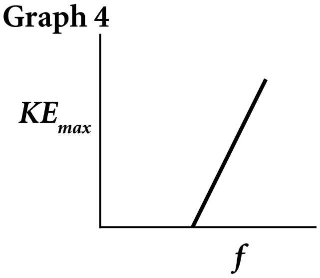
Graph 3



Graph 2



Graph 4

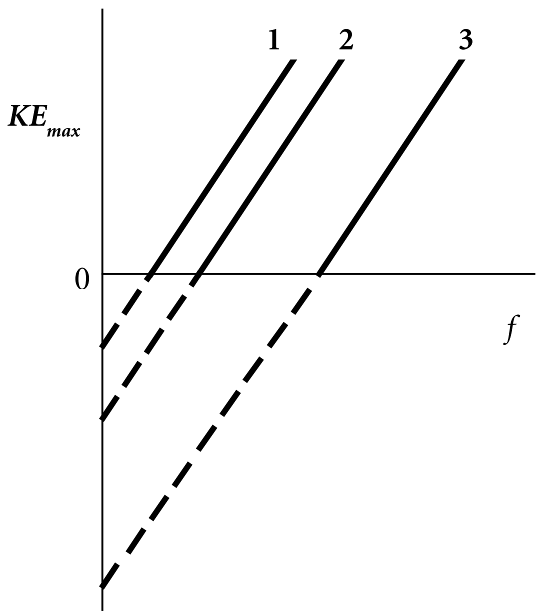


Solution

Graph **4** shows the correct relationship between KE_{max} and frequency.

Exercise 3

The graph below shows the maximum kinetic energy of the photoelectrons for three different metals versus the frequency of the light. Use the graph to derive an equation for the energy (KE_{max}) in terms of the frequency (f).



Solution

Straight-line equation:

$y = mx + b$,

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{\Delta KE}{\Delta f} = h\,.$$

The y-intercept = $-KE_o$ or $-hf_o$.

Substituting:

$KE_{max} = hf - hf_o$.

Exercise 4

The experimental data of a photoelectric effect experiment using cesium are given in the table.

Stopping Potential (V)	Frequency (Hz)
0.1	5.49×10^{14}
0.7	6.88×10^{14}
1.3	8.21×10^{14}
2.7	11.8×10^{14}

a. Graph the stopping potential versus the frequency.

b. Use the graph to find the threshold frequency for cesium.

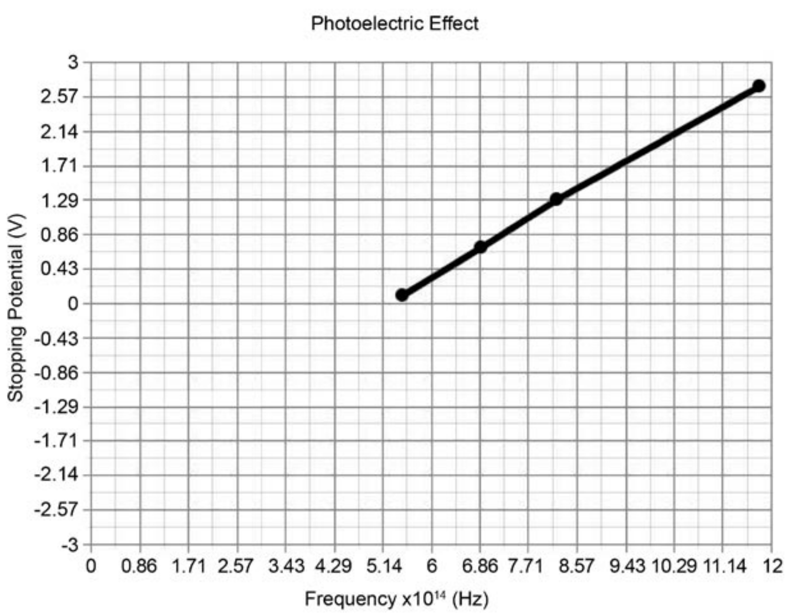
c. Use the graph to determine Planck's constant.

d. Calculate the percent error of the experimental value if the accepted value for Planck's constant is 6.63×10^{-34} J·s.

e. Use the graph to find the work function of the metal. Give your answer in joules.

Solution

a.



b. The threshold frequency f_o is the intercept with the x-axis, and it can be found simply by extrapolating the line:

$f_o \approx 5.18 \times 10^{14}$ Hz.

c. Planck's constant can be determined from the slope of the graph:

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{1.3\text{ V} - 0.1\text{ V}}{8.21 \times 10^{14}\text{ Hz} - 5.49 \times 10^{14}\text{ Hz}} = 4.41\times10^{-15}\text{eV}\cdot\text{s}.$$

d. Planck's constant needs to be converted to J·s:

$$\left(4.41\times10^{-15}\text{ eV}\cdot\text{s}\right)\left(1.6\times10^{-19}\text{ J/eV}\right) = 7.06\times10^{-34}\text{ J}\cdot\text{s}.$$

Percent error:

$$\%\text{ error} = \left|\frac{h_{exp} - h_{theor}}{h_{theor}}\right|(100),$$

$$\%\text{ error} = \left|\frac{7.06\times10^{-34}\text{ J}\cdot\text{s} - 6.63\times10^{-34}\text{J}\cdot\text{s}}{6.63\times10^{-34}\text{ J}\cdot\text{s}}\right|(100) = 6.5\%.$$

e. The work function W_o is the intercept with the y-axis, and it can be found simply by extrapolating the line:

$W_o \approx 2.14$ eV or 3.4×10^{-19} J.

Contributors

About the Editor

Dolores Gende has an undergraduate degree in chemical engineering from the Iberoamericana University in Mexico City. She has been teaching science and math for over 20 years in the United States and other countries, including Mexico, Belgium, and the Netherlands Antilles. She has 13 years of experience teaching college-level introductory physics courses and presently teaches at the Parish Episcopal School in Dallas. In March 2006, she received the Excellence in Pre-College Physics Teaching Award from the Texas Section of the American Association of Physics Teachers. She serves as an AP Physics Table Leader, an AP Physics workshop consultant, and as the AP Physics content adviser for AP Central, for which she has written five feature articles and several reviews for the site's "Teachers' Resources" section. Her award-winning physics Web site (http://dgende.homestead.com) is an important reference tool for teachers across the country.

Laurence S. Cain is professor of physics and chair of the Physics Department at Davidson College, where he has taught since 1978. He received his B.S. from Wake Forest University and his M.S. and Ph.D. from the University of Virginia. His research interests are in condensed matter physics, particularly the elastic, mechanical, optical, and defect properties of solids. He has produced numerous research papers, contributed papers with his undergraduate students, and participated in the development of interactive curricular materials for physics instruction. He is currently treasurer of the Southeastern Section of the American Physical Society and chair of the AP Physics Development Committee. He is a recipient of the Thomas Jefferson Award, the Omicron Delta Kappa (ODK) Teaching Award, and the Matthews Travel Award from Davidson College.

Hasan Fakhruddin is an instructor at the Indiana Academy for Science, Mathematics, and Humanities in Muncie, Indiana, where he has taught AP Physics B and C for 15 years. A member of the AP Physics Development Committee for four years, he has attended the AP Physics Reading for eight years. He has published a number of articles in the *Physics Teacher* and other journals, has given presentations at NSTA (National Science Teachers Association) on AP Physics courses, and has written two articles for the AP Physics section of AP Central.

Boris Korsunsky holds graduate degrees in physical chemistry and physics from colleges in Moscow and a doctorate in learning and teaching from Harvard Graduate School of Education. He has been teaching high school physics for almost two decades and has been involved with AP Physics since 1994. He has published numerous articles on the pedagogy of physics education and two books of original problems. A former coach of the U.S. Physics Team, he currently teaches at Weston High School in Massachusetts and conducts teacher workshops.

James Mooney has been teaching AP Physics B and C for nearly 30 years, the last 20 at the Taft School in Connecticut. He has led summer workshops on the AP Physics curriculum, and has participated in the AP Reading for five years. His study guides for AP Physics, *apadvantage™ Physics B* and *apadvantage™ Physics C*, were recently published by Peoples Education.

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